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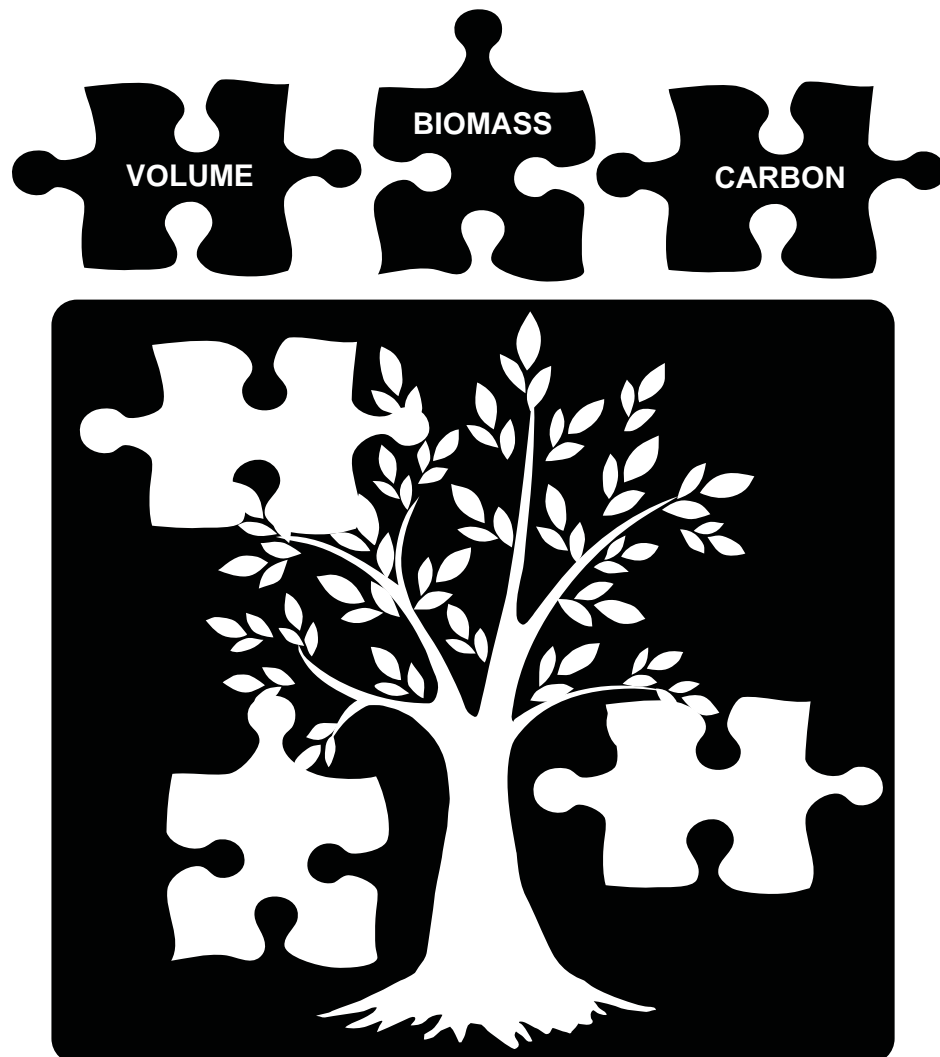
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Methods and Equations for Estimating Aboveground Volume, Biomass, and Carbon for Trees in the U.S. Forest Inventory, 2010

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Abstract

The U.S. Forest Service, Forest Inventory and Analysis (FIA) program uses numerous models and associated coefficients to estimate aboveground volume, biomass, and carbon for live and standing dead trees for most tree species in forests of the United States. The tree attribute models are coupled with FIA's national inventory of sampled trees to produce estimates of tree growing-stock volume, biomass, and carbon, which are available in the Forest Inventory and Analysis database (FIADB; <http://fiatools.fs.fed.us>). To address an increasing need for accurate and easy-to-use documentation of relevant tree attribute models, needed individual tree gross volume, sound volume, biomass (including components), and carbon models for species in the United States are compiled and described in this publication with accompanying electronic files on a CD-ROM included with the publication. This report describes models currently in use as of 2010. These models are subject to change as the FIADB and associated tree attribute models are improved.

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INTRODUCTION

The official annual U.S. greenhouse gas (GHG) inventories contain forest carbon and other greenhouse gas estimates (e.g., see U.S. Environmental Protection Agency 2010) based on U.S. Forest Service's, Forest Inventory and Analysis (FIA) program survey data, coupled with other basic models (Heath et al. 2010). The FIA program, through its regional units (Northern, Southern, Rocky Mountain, and Pacific Northwest; see USDA For Serv. 2011), is the primary provider of state-, regional-, and national-scale forest population estimates such as forest land area, and tree volume, growth, mortality, and removals. U.S. forest carbon inventories have used biomass estimates based on equations from Jenkins et al. (2003 and 2004), due to previous inconsistencies in tree biomass estimates between FIA regions (pre-2009). Estimates for a tree species in one region may have differed notably from estimates of the same species in another region. In 2009, FIA adopted a process for estimating tree biomass (Heath et al. 2009). This methodology is now consistently applied across the United States, with estimates reported in the publicly available Forest Inventory and Analysis database (FIADB; USDA For. Serv. 2010a).

With emerging carbon markets and growing interest in bioenergy at various scales across the United States, many individuals are interested not only in the models FIA uses for individual tree species, but also in adjusting models and associated coefficients for their own population estimates. The purpose of this report is to present in an accessible format the methodology and requisite equations for estimating attributes for most of the tree species in the United States. Until publication of this report, similar information had been only available for the southern (Oswalt and Conner 2011) and northern (Miles and Hill 2010) regions of the eastern United States. The volume and biomass attributes described in this paper and the conditions under which they are computed are described in the following tabulation. Carbon estimates follow directly from biomass estimates based on the assumption that biomass is 50 percent carbon (Birdsey 1992).

The process of tree attribute estimation for all sample trees 5 inches in diameter and larger begins with the estimation of tree gross volume (VOLCFGRS) and tree sound volume (VOLCFSND). For the majority of tree species (non-woodland trees where diameter is measured at 4.5 feet above ground; refer to REF_SPECIES.xlsx for woodland species designation) biomass and carbon by component (stump, bole and tops) are also estimated (Table 1). For all woodland trees (trees where diameter is measured at the root collar) biomass and carbon are estimated but are not broken down by component. For all sample trees between 1 to 4.9 inches in diameter only total biomass and carbon are estimated. Estimates of volume (VOLCFGRS and VOLCFSND) and breakdowns of total biomass and carbon into components are not made for trees < 5.0 inches in diameter. No estimates of volume, biomass, or carbon are made for sample trees where diameter is < 1.0 inch.

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Table 1.—Volume and biomass components estimated by FIA for live and standing dead trees

	Non-woodland species: Diameter is measured at 4.5 feet above ground	Woodland species: Diameter is measured at root collar
Less than 1.0 inches in diameter	No estimates of volume or biomass	
1.0 to 4.9 inches in diameter	No estimate of volume	
	Sapling biomass = Total aboveground biomass	Biomass of woodland species = Total aboveground biomass
5.0 inches in diameter and larger	Gross volume Sound volume (Stump wood biomass + Stump bark biomass + Bole wood biomass + Bole bark biomass + Top and branch biomass) = Total aboveground biomass	Gross volume Sound volume Biomass of woodland species = Total aboveground biomass

Biomass for woodland species < 5.0 inches in diameter and saplings (non-woodland species between 1.0 to 4.9 inches in diameter) are estimated directly from the biomass equation in Jenkins et al. (2003 and 2004) using the observed diameter and a sapling adjustment factor. Biomass for woodland species ≥ 5.0 inches diameter is based on volume estimates for the entire aboveground portion of the sample tree with no component breakdown.

Volume, biomass, and carbon models for tropical trees (e.g., species in Puerto Rico or Hawaii) are not included in this report. This compilation does not necessarily reflect the ideal system for estimating an array of tree attributes (e.g., volume and carbon) that are additive among individual tree components (i.e., scalable tree attributes from components to whole tree) and consistent across diverse forest conditions at a national scale (i.e., do not arbitrarily change at political boundaries). Rather, this system reflects an effort to meld localized tree volume/biomass information into a consistent, national approach. We suggest that future research focus on consistent national-scale individual tree volume/biomass/carbon equations that accommodate the diversity of tree habitats (e.g., woodlands or tropics) and conditions (e.g., sound or rotten).

METHODS FOR BIOMASS ESTIMATION

The approach used for nationally consistent biomass estimation of non-sapling and non-woodland trees in the FIADB is called the component ratio method (CRM; Heath et al. 2009). CRM entails 1) measuring attributes of the tree in the field; 2) applying those tree measurements to the applicable volume model to compute both gross cubic-foot volume (VOLCFGRS) and sound cubic-foot volume (VOLCFSND) of wood in the bole; 3) converting the sound cubic-foot volume (VOLCFSND) of wood in the bole to mass and estimating bark biomass using compiled sets of specific gravity (Miles and Smith 2009); 4) calculating the biomass of tops and limbs as a proportion of the bole based on component proportions from Jenkins et al. (2003); 5) calculating stump volume based on equations in Raile (1982) and converting to biomass, and 6) summing all aboveground components for a total aboveground biomass estimate.

The use of CRM requires the calculation of VOLCFSND. Because VOLCFSND is based on gross cubic-foot volume (VOLCFGRS) for many areas of the country, VOLCFGRS must also be calculated. It should be noted that FIA calculates many types of volumes of interest to a range of user groups including: total stem volumes (calculated for all trees ≥ 1.0 -inch d.b.h.), cubic-foot volumes up to various top diameters (e.g., 4-, 6-, or 8-inch tops), and board-foot volumes using Scribner or International $\frac{1}{4}$ inch volume rules (for definitions see Helms 1998). The CRM is based on VOLCFGRS and VOLCFSND; only those volume equations are compiled in this study (Appendix A).

The five general steps for calculation of tree (d.b.h. ≥ 5.0 inches) oven-dry mass (lb) are:

- 1) Obtain needed tree and plot or condition measurements using FIA protocols (e.g., tree diameter, rotten and missing cull, site index, height, basal area)
- 2) Calculate VOLCFGRS
- 3) Calculate VOLCFSND (i.e., subtract rotten or missing from VOLCFGRS)
- 4) Convert VOLCFSND to bole biomass
- 5) Calculate additional biomass components (e.g., tops and limbs, stump) that are summed for total biomass

Note that estimation procedures are different for saplings (d.b.h. ≥ 1.0 inch and < 5.0 inches). Sapling biomass is based on biomass computed from Jenkins et al. (2004) using the observed diameter and an adjustment factor—computed as a national average ratio of the CRM total biomass divided by the Jenkins et al. (2003 and 2004) total biomass for all 5.0-inch trees, which is the size at which biomass based on volume begins (Heath et al. 2009). At this time estimation procedures are not different for standing dead trees. Users of this documentation should consider adopting decay and structural reduction factors for standing dead trees such as those currently under development by FIA (Domke et al. in press). Such information could reduce any potential upward bias of estimating standing dead wood volume/biomass using live tree procedures.

The steps for biomass calculation are described for each component separately in the Biomass Calculations section of this report. We also provide example calculations for a live growing-

stock tree, a live rough cull tree, a standing dead tree, and a small diameter tree for comparison and reference. The text refers to appendices where volume equations are presented; coefficients for the equations and species-specific information can be found in the companion spreadsheets (volcfgrs_eqn_coefs.xlsx, volcfsnd_eqn_coefs.xlsx, and REF_SPECIES.xlsx) on the CD included in this report.

Tree Measurements

Field protocols for FIA are found in USDA For. Serv. (2010b). The diameter of most tree species is measured at d.b.h., except in the case of woodland tree species where it is measured at d.r.c. Species information, including species names, codes, diameter type, wood and bark specific gravity, bark percentage, sapling adjustment factors, Jenkins et al. (2004) biomass coefficients, and Raile (1982) stump volume coefficients can be found in the REF_SPECIES table in FIADB, the original publications cited in this document, or on the CD included with this report. Models may differ by state or ecoregion within a state, thus location of the tree is important. See Figure 1 for definition of areas for model determination. Other information that may be needed includes site index, height, bole height, percent of cubic-foot volume rotten or missing, percent of sound dead cull as a percentage of merchantable bole, and number of stems (for trees measured at d.r.c.). For standing dead trees, a decay class is determined for each tree, but is not currently used in estimation procedures (USDA For. Serv. 2010b).

Gross Volume Estimation

Gross cubic-foot volume is defined as “the total volume inside bark of the central stem of a tree ≥ 5.0 inches d.b.h. from a 1-foot stump to a minimum 4-inch top diameter outside bark (d.o.b.), or to where the central stem breaks into limbs all of which are less than 4.0 inches d.o.b.” (Woudenberg et al. 2010). Gross volume includes rotten and missing parts and form cull. Rotten and missing cubic-foot cull volume is estimated to the nearest 1 percent in the field. This estimate does not include any cull deduction above actual length so volume lost from a broken top is not included. Form cull is the percent of the cubic-foot volume that is cull due to form defect, and is only collected at some locations. Gross volume is set to null (not available) if d.b.h. is not available. Usually, if d.b.h. is < 5.0 inches, gross volume is set to null, with the possible exception of woodland tree species (i.e., d.r.c.) in Rocky Mountain and Southern States.

The form of the models to calculate VOLCFGRS are listed by FIA unit (i.e., Northern or Southern) and sub-unit model region (i.e., eastern MT or western MT; Fig. 1) in Appendices A and B (see CD for model coefficients), along with the reference for each model. Unless otherwise noted, the **b** values in these tables are regression parameter estimates and the **x** values are the observed attributes. The bold text indicates conditions that should be met regarding the observed attributes. The values for the coefficients are given in the VOLCFGRS spreadsheet provided on a CD in this report. To find the appropriate model and coefficients for the tree species and location, use the following steps:

- 1) To calculate VOLCFGRS, locate the species-specific equation for the region of interest found in Tables 1-4 of Appendix A and Table 6 of Appendix B.
- 2) Find species-specific coefficients (denoted by the letter **b**, as in b_1 , b_2 , etc.) in the spreadsheet volcfgrs_eqn_coefs.xlsx.

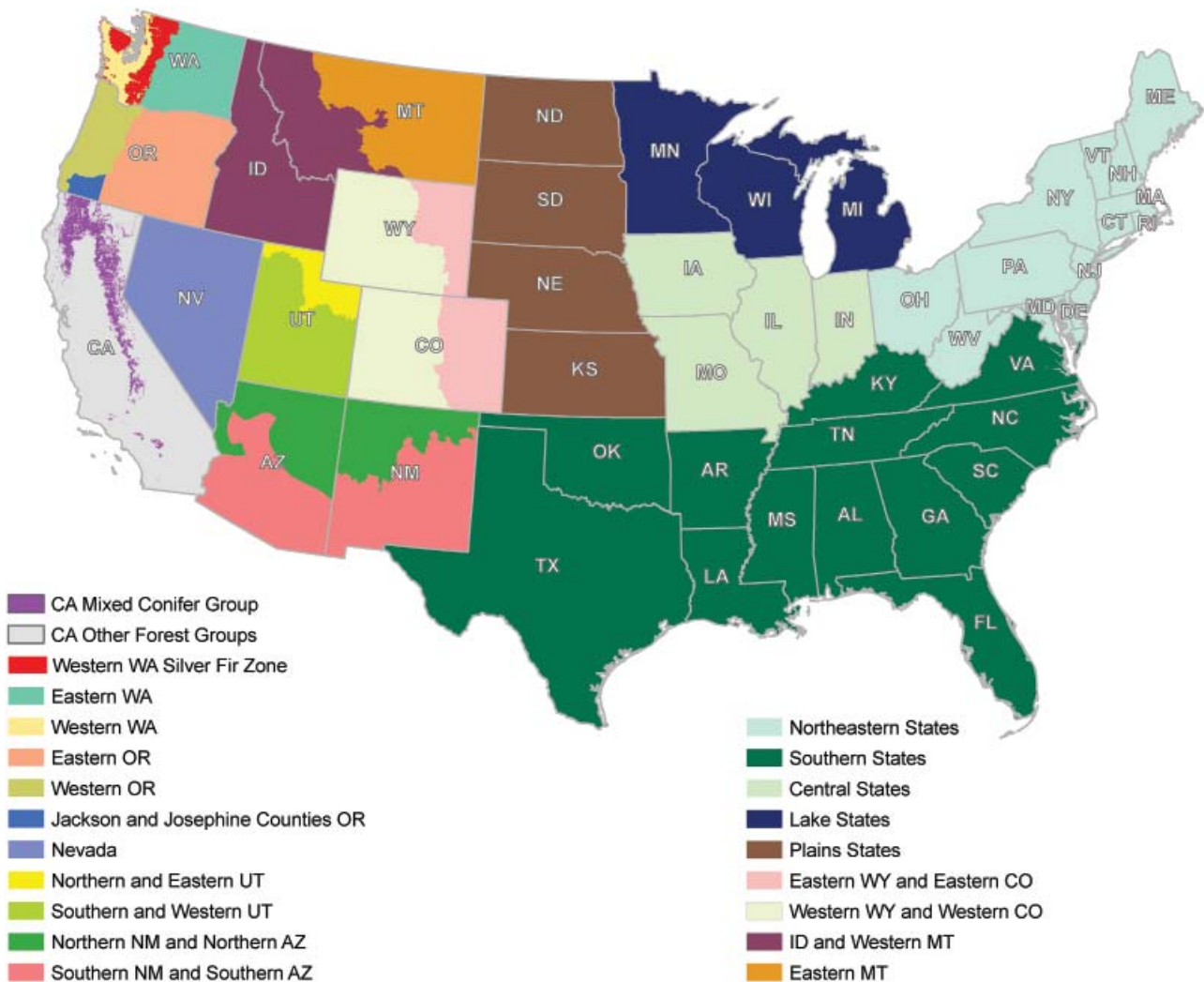


Figure 1.—Delineated areas for identification of gross cubic-foot growing stock volume models and coefficients from the spreadsheet file. Further description is given in U.S. Forest Service (2010b; Appendix c).

- 3) In the `volcfrs_eqn_coefs` spreadsheet, locate the regional config worksheet for the area of interest. They are designated as follows: NCCS – North Central Central States; NCLS – North Central Lake States; NCPS – North Central Plains States; NE – Northeastern States; AZ_N_NM_N – northern Arizona and northern New Mexico; AZ_S_NM_S – southern Arizona and southern New Mexico; CO_E_WY_E – eastern Colorado and eastern Wyoming; CO_W_WY_W – western Colorado and western Wyoming; ID_MTW – Idaho and western Montana; MT_E – eastern Montana; NV – Nevada; UT_NE – northern and eastern Utah; UT_SW – southern and western Utah; S – Southern States; CA – California (except mixed conifer forest type); CA_MC – California mixed conifer forest type; OR_E – eastern Oregon; OR_WA – western Oregon; OR_JJ – Oregon (Jackson and Josephine Counties); WA_E – eastern Washington; WA_W – western Washington; and WA_CF – Washington (high-elevation silver fir region).
- 4) Find the row corresponding to the particular species; the sheet is ordered by species number (Column C). For a list of all species with common and scientific names, refer to the worksheet `ref_species_reduced`. The column `COEF_TABLE` contains the name

of the table which has the coefficients for this species. The column COEF_TBL_SP contains the species number you should look for in the table.

- 5) Insert the appropriate coefficients and the tree-specific variables into the equation from step 1 to estimate VOLCFGRS.

Sound Volume Estimation

Sound cubic-foot volume is the volume of sound wood in the central stem of a tree ≥ 5.0 inches diameter from a 1-foot stump to a minimum 4-inch top d.o.b., or to where the central stem breaks into limbs, all of which are less than 4.0 inches d.o.b. Note that in some instances VOLCFSND may be 0 or null and the VOLCFGRS is > 0 in FIADB. In those cases, biomass is calculated using VOLCFGRS adjusted for the percentage sound. When both VOLCFSND and VOLCFGRS are 0 or null, biomass is calculated using net cubic-foot volume (VOLCFNET) adjusted for the percentage sound (Woudenberg et al. 2010).

The form of the models to calculate VOLCFSND from VOLCFGRS are listed by FIA region in Table 5 of Appendix A (see CD for model coefficients; see Appendix C for Alaska VOLCFSND algorithm), along with the reference for the model. Unless noted, the b values in these tables are regression parameter estimates and the x values are the observed attributes. The bold text indicates conditions that should be met regarding the observed attributes. To find the appropriate model and coefficients for the tree species and location, use the following steps:

- 1) To calculate VOLCFSND, locate the appropriate equation in Appendix A, Table 5 (see Appendix C for Alaska VOLCFSND algorithm). Only a few of the equations use species-specific coefficients (denoted by the letter b , as in b_1 , b_2 , etc.). If the applicable volume equation you want to use contains these coefficients, they may be found in the Excel spreadsheet `volcfsnd_eqn_coefs.xlsx`
- 2) In the `volcfsnd_eqn_coefs` spreadsheet, locate the regional config worksheet for the area of interest. They are designated as follows: NCCS – North Central Central States; NCLS – North Central Lake States; and NCPS – North Central Plains States. The column COEF_TABLE contains the name of the table containing the coefficients for this species. The column COEF_TBL_SP contains the species number you should look for in the table
- 3) Insert the appropriate coefficients (if present), the tree-specific variables and the VOLCFGRS (calculated in step 5 under Gross Volume Estimation) into the equation from step 1 in this section to estimate VOLCFSND.

Biomass Calculations

Biomass of bole

Biomass in the bole of a species is calculated by multiplying the VOLCFSND by weight of 1 cubic foot of water (62.4 lb) to convert volume to mass, and then multiplying by the specific gravity of wood and bark, separately for the species listed in Miles and Smith (2009). For a complete list of specific gravities for all tree species in FIADB, please refer to REF_SPECIES.xlsx included in this publications's CD-ROM. The specific gravities of bark and wood are different.

The bark component includes an additional term, bark as a proportion of wood volume, so the two components must be calculated separately and then summed.

Bole wood biomass:

$$B_{odw} = V_{gw} * SG_{gw} * W \quad (\text{eq. 1})$$

where

B_{odw} = oven-dry biomass (lb) of wood

V_{gw} = VOLCFSND of green wood in the central stem

SG_{gw} = basic specific gravity of wood (oven-dry mass of green volume)

W = weight of ft³ of water (62.4 lb)

Bark biomass:

$$B_{odb} = V_{gw} * BV\% * SG_{gb} * W \quad (\text{eq. 2})$$

where

B_{odb} = oven-dry biomass (lb) of bark

V_{gw} = VOLCFSND of green wood in the central stem

$BV\%$ = bark as a percentage of wood volume

SG_{gb} = basic specific gravity of bark (oven-dry mass of green volume)

W = weight of ft³ of water (62.4 lb)

Total bole (bole and bark) biomass (DRYBIO_BOLE):

$$B_{odt} = B_{odw} + B_{odb} \quad (\text{eq. 3})$$

where

B_{odw} = oven-dry biomass (lb) of wood

B_{odb} = oven-dry biomass (lb) of bark

B_{odt} = total oven-dry bole biomass (lb) in wood and bark

Biomass of tree components

Biomass in the stumps and tops and limbs of trees ≥ 5.0 inches d.b.h. is estimated as a proportion of CRM bole biomass (eq. 3) using component ratios from Jenkins et al. (2004) and Raile (1982). All tree components derived from Jenkins et al. (2004) and Raile (1982) must be multiplied by an adjustment factor to estimate CRM biomass.

CRM adjustment factor:

$$CRM_{AdjFac} = B_{odt}/MST \quad (\text{eq. 4})$$

where

CRM_{AdjFac} = component ratio method adjustment factor for tree components derived from Jenkins et al. (2003) and Raile (1982)

B_{odt} = total oven-dry biomass (lb) (eq. 3)

MST = merchantable oven-dry bole biomass (lb) (Jenkins et al. 2004)

Biomass in the stump of a species is estimated using volume equations developed by Raile (1982), converting volume to mass by multiplying by the specific gravity of wood and bark (Miles and Smith 2009) and by 1 cubic foot of water (62.4 lb), and finally multiplying by the CRM adjustment factor (eq. 4).

Stump volume:

$$S_{vosb \text{ or } visb} = \frac{\pi(dbh)^2}{4(144)} \left[\left((A - B)^2 h + 11B(A - B)\ln(h + 1) - \frac{30.25}{h + 1} B^2 \right) \right] \frac{b}{a} \quad (\text{eq. 5})$$

where

$S_{vosb \text{ or } visb}$ = stump volume inside bark (*visb*) or outside bark (*vosb*) (ft³)

A = Coefficient (species parameter) from Table 2 in Raile (1982)

B = Coefficient (species parameter) from Tables 1 and 2 in Raile (1982)

h = height above ground (ft)

ln = natural logarithm

a = lower stump height (ft) – 0 ft in FIADB

b = upper stump height (ft) – 1 ft in FIADB

For stump volume estimates outside (*vosb*) the bark, A should be set to 1 and the species parameter (B) in Table 1 should be used. For stump volume estimates inside the bark (*visb*), Table 2 in Raile (1982) should be used for both coefficients.

Stump wood biomass:

$$S_{odsw} = S_{visb} * SG_{gw} * W \quad (\text{eq. 6})$$

where

S_{odsw} = oven-dry biomass (lb) of stump wood

S_{visb} = stump volume inside the bark (eq. 5)

SG_{gb} = basic specific gravity of wood (oven-dry mass of green volume)

W = weight of ft³ of water (62.4 lb)

Stump bark biomass:

$$S_{odsb} = (S_{vosb} - S_{visb})SG_{gb} * W \quad (\text{eq. 7})$$

where

S_{odsb} = oven-dry biomass (lb) of stump bark

S_{vosb} = volume (ft³) of stump outside the bark (eq. 5)

S_{visb} = volume (ft³) of stump inside the bark (eq. 5)

SG_{gb} = basic specific gravity of bark (oven-dry mass of green volume)

W = weight of ft³ of water (62.4 lb)

Total stump (wood and bark) biomass (DRYBIO_STUMP):

$$S_{odt} = (S_{odsw} + S_{odsb})CRM_{AdjFac} \quad (\text{eq. 8})$$

Where

S_{odt} = oven-dry stump biomass (lb)

B_{odw} = oven-dry biomass (lb) of wood (eq. 6)

B_{odb} = oven-dry biomass (lb) of bark (eq. 7)

CRM_{AdjFac} = component ratio method adjustment factor (eq. 4)

The tops and limbs component ratio is calculated by subtracting the total aboveground biomass estimate from Jenkins et al. (2004) from the aboveground biomass components for that species in Jenkins et al. (2004)—foliage, stem bark and wood, and stump (from Raile 1982)—and multiplying by the CRM adjustment factor (eq. 4).

Top and branch biomass:

$$T_{odt} = (TAB - MST - STP - FOL)CRM_{AdjFac} \quad (\text{eq. 9})$$

where

T_{odt} = oven-dry biomass (lb) of top and branches

TAB = total aboveground oven-dry biomass (lb) (Jenkins et al. 2004)

MST = merchantable stem oven-dry biomass (lb) (Jenkins et al. 2004)

STP = stump oven-dry biomass (lb) (Raile 1982)

FOL = foliage oven-dry biomass (lb) (Jenkins et al. 2004)

CRM_{AdjFac} = component ratio method adjustment factor (eq. 4)

Biomass of saplings

Biomass of saplings (trees < 5.0 inches d.b.h.) is based on estimates from Jenkins et al. (2004) on the observed diameter multiplied by a sapling adjustment factor. The adjustment factors currently used are computed as a national average ratio of the CRM total biomass divided by the total biomass from Jenkins et al. (2004) for all 5.0-inch trees, which is the size at which biomass-based volume begins. Each species group has an adjustment factor listed in Heath et al. (2009), and which can be found in the REF_SPECIES file on the CD.

Sapling biomass (DRYBIO_SAPLING):

$$Sap_{odt} = (TAB - FOL)Sap_{adj} \quad (\text{eq. 10})$$

where

Sap_{odt} = oven-dry sapling biomass (lb)

TAB = total aboveground oven-dry biomass (lb) (Jenkins et al. 2004)

FOL = foliage oven-dry biomass (lb) (Jenkins et al. 2004)

Sap_{adj} = sapling adjustment factor (Heath et al. 2009)

Biomass of woodland species (DRYBIO_WDLD_SPP)

Woodland species are defined as those sample trees where diameter is measured at root collar (Woudenberg et al. 2010). Woodland species biomass for stems < 5.0 inches d.r.c. is estimated in the same way as sapling biomass is estimated. Woodland species \geq 5.0 inches in diameter have a volume estimate from ground to tip, which is used to estimate total aboveground biomass in the same way bole biomass is calculated.

Biomass of standing dead trees

Standing dead biomass is computed in the same manner as live tree biomass. Standing dead trees, like live trees, can have rotten and missing cull which is deducted when converting VOLCFGRS to VOLCFSND. Future research into decay and structural reductions based on a tree's decay class is suggested as a way to reduce potential overestimates of volume/biomass (Domke et al. in press).

EXAMPLES

We present example calculations for using the CRM to estimate aboveground biomass and carbon for 10.5-inch growing stock, rough cull, and standing dead quaking aspen trees and a 4.5-inch quaking aspen sapling in the North Central - Lake States (NCLS) region. Each example reflects actual sample trees in the FIADB. Species information including specific gravity values for wood and bark, bark percentage estimates, sapling adjustment factors and biomass model coefficients are found in the publications cited in this report, FIADB in the REF_SPECIES table, or in the REF_SPECIES.xlsx workbook on CD. Each biomass equation used in the examples is referenced back to the original text description in this document where coefficients are described and the original citation is provided. Note that values in these examples may vary slightly from FIADB values due to rounding.

Example 1: Volume, biomass, and carbon calculations for a live growing-stock tree

Example tree information: live (STATUSCD = 1) growing stock (TREECLCD = 2) quaking aspen (SPCD = 746) that is 10.5 inches d.b.h.

1. Gross volume (VOLCFGRS)

V_1 = top diameter outside bark = 4 inches

$$V_2 = \text{predicted bole length} = b_{12} + b_{13} (1 - e^{-b_{14}x_1})^{b_{15}} x_2^{b_{16}} (b_{17} - \frac{V_1}{x_1})^{b_{18}} x_3^{b_{19}} = 4.5 \\ + 6.4301(1 - e^{-0.23545*10.5})^{1.338} * 66^{0.4737}(1.00001 - 4.5/10.5)^{0.73385} * 72.53^{0.08228} = \\ 46.11 \text{ ft}$$

x_1 = d.b.h. = 10.5 inches

x_2 = site index = 66 ft

x_3 = basal area of live trees on plot/condition = 72.53 ft²

$$\text{VOLCFGRS} = (b_0 + b_1x_1 + b_2V_1 + b_3x_1^2 + b_4x_1^2V_2 + b_5V_2^2 + b_6V_2V_1^2 + b_7x_1^2V_2^3 + \\ b_8x_1^2V_2^2V_1)b_9(b_{10} + b_{11}x_1) / 100 = (0.0030086 + 0.0020355*10.5 + -0.0030018*4 \\ + 0.000062381*10.52 + 0.000025705*10.52*46.11 + -0.000007009*46.112 + \\ 0.000036708*46.11*42 + 0.000000000814*10.52*46.113 + -0.0000000019*10.52*4 \\ 6.112*4)79(91.51 + 0.37*10.5) / 100 = 12.74 \text{ ft}^3$$

2. Sound volume (VOLCFSND)

$$\text{VOLCFSND} = \text{VOLCFGRS}(1 - \frac{b_1b_2}{100}) = 12.74(1 - \frac{5.2*0.1}{100}) = 12.68 \text{ ft}^3$$

3. Bole biomass (DRYBIO_BOLE)

Bolewood biomass (eq. 1): $B_{odw} = V_{gw} * SG_{gw} * W = 12.68 * 0.35 * 62.40 = 276.89 \text{ lb}$

Bole bark biomass (eq. 2): $B_{odb} = V_{gw} * BV\% * SG_{gb} * W = 12.68 * 0.144 * 0.5 * 62.4 \\ = 56.96 \text{ lb}$

Total bole biomass (eq. 3): $B_{odt} = B_{odw} + B_{odb} = 276.89 + 56.96 = 333.85 \text{ lb}$

4. Component ratio method adjustment factor (CRM_{AdjFac})

$$\text{CRM adjustment factor (eq.4): } \text{CRM}_{\text{AdjFac}} = \frac{B_{odt}}{MST} = \frac{333.85}{444.87} = 0.75$$

5. Stump volume and biomass (DRYBIO_STUMP)

$$\text{Stump volume (eq. 5): } S_{vosb} = \frac{\pi(abh)^2}{4(144)} \left[\left[(A - B)^2 h + 11B(A - B)\ln(h + 1) - \frac{30.25}{h+1} B^2 \right] \right] \frac{b}{a} = \\ \frac{\pi(10.5)^2}{4(144)} \left[\left[(1 - 0.09658)^2 1 + 11 * 0.09658(1 - 0.09658)\ln(1 + 1) - \frac{30.25}{1+1} 0.09658^2 \right] - \right. \\ \left. \left[(1 - 0.09658)^2 0 + 11 * 0.09658(1 - 0.09658)\ln(0 + 1) - \frac{30.25}{0+1} 0.09658^2 \right] \right] = 0.97565 \text{ ft}^3$$

$$S_{visb} = \frac{\pi(10.5)^2}{4(144)} \left[\left((0.91882 - 0.08593)^2 1 + 11 * 0.08593(0.91882 - 0.08593)\ln(1 + 1) - \frac{30.25}{1+1} 0.08593^2 \right) - \left((0.91882 - 0.08593)^2 0 + 11 * 0.08593(0.91882 - 0.08593)\ln(0 + 1) - \frac{30.25}{0+1} 0.08593^2 \right) \right] = 0.81243 \text{ ft}^3$$

Stump wood biomass (eq. 6): $S_{odsw} = S_{visb} * SG_{gw} * W = 0.81243 * 0.35 * 62.4 = 17.74 \text{ lb}$

Stump bark biomass (eq. 7): $S_{odsb} = (S_{vosb} - S_{visb})SG_{gb} * W = (0.97565 - 0.81243)0.5 * 62.4 = 5.09 \text{ lb}$

Total stump biomass (eq. 8): $S_{odt} = (S_{odsw} + S_{odsb})CRM_{AdjFac} = (17.74 + 5.09)0.75 = 17.14 \text{ lb}$

6. Top and branch biomass (DRYBIO_TOP)

Top and branch biomass (eq. 9): $T_{odt} = (TAB - MST - STP - FOL) CRM_{AdjFac} = (612.75 - 444.87 - 22.84 - 12.90)0.75 = 99.17 \text{ lb}$

7. Total aboveground biomass (CRM)

Total aboveground biomass = $B_{odt} + S_{odt} + T_{odt} = 333.85 + 17.14 + 99.17 = 450.16 \text{ lb}$

8. Total aboveground carbon (CARBON_AB)

Total aboveground carbon = $(B_{odt} + S_{odt} + T_{odt}) * 0.5 = (333.85 + 17.14 + 99.17)0.5 = 225.08 \text{ lb}$

Example 2: Volume, biomass, and carbon calculations for a live rough cull tree

Example tree information: live (STATUSCD = 1) rough cull (TREECLCD = 3) quaking aspen (SPCD = 746) sample tree that is 10.5 inches d.b.h.

1. Gross volume (VOLCFGRS)

$V_1 = \text{top diameter outside bark} = 4 \text{ inches}$

$V_2 = \text{predicted bole length} = b_{12} + b_{13} (1 - e^{-b_{14}x_1})^{b_{15}} x_2^{b_{16}} (b_{17} - \frac{V_1}{x_1})^{b_{18}} x_3^{b_{19}} = 4.5 + 6.4301(1 - e^{-0.23545*10.5})^{1.338} * 73^{0.4737}(1.00001 - 4.5/10.5)^{0.73385} * 82.27^{0.8228} = 48.59 \text{ ft}$

$x_1 = \text{d.b.h.} = 10.5 \text{ inches}$

$x_2 = \text{site index} = 73 \text{ ft}$

$x_3 = \text{basal area of live trees on plot/condition} = 82.27 \text{ ft}^2$

$VOLCFGRS = (b_0 + b_1x_1 + b_2V_1 + b_3x_1^2 + b_4x_1^2V_2 + b_5V_2^2 + b_6V_2V_1^2 + b_7x_1^2V_2^3 + b_8x_1^2V_2^2V_1)b_9(b_{10} + b_{11}x_1) / 100 = (0.0030086 + 0.0020355*10.5 + -0.0030018*4 + 0.000062381*10.5^2 + 0.000025705*10.5^2*48.59 + -0.000007009*48.59^2 + 0.000036708*48.59*4^2 + 0.00000000814*10.5^2*48.59^3 + -0.0000000019*10.5^2*48.59^2*4)79(91.51 + 0.37*10.5) / 100 = 13.36 \text{ ft}^3$

2. Sound volume (VOLCFNSND)

$VOLCFNSND = VOLCFGRS(1 - \frac{b_1b_2}{100}) = 13.36(1 - \frac{21.8*0.15}{100}) = 12.92 \text{ ft}^3$

3. Bole biomass (DRYBIO_BOLE)

Bolewood biomass (eq. 1): $B_{odw} = V_{gw} * SG_{gw} * W = 12.92 * 0.35 * 62.40 = 282.17 \text{ lb}$

Bole bark biomass (eq. 2): $B_{odb} = V_{gw} * BV\% * SG_{gb} * W = 12.92 * 0.144 * 0.5 * 62.4 = 58.05 \text{ lb}$

Total bole biomass (eq. 3): $B_{odt} = B_{odw} + B_{odb} = 282.17 + 58.05 = 340.30$ lb

4. Component ratio method adjustment factor (CRM_{AdjFac})

$$CRM_{AdjFac} = \frac{B_{odt}}{MST} = \frac{340.30}{444.87} = 0.76$$

5. Stump volume and biomass (DRYBIO_STUMP)

$$S_{vosb} = \frac{\pi(dbh)^2}{4(144)} \left[\left((A - B)^2 h + 11B(A - B)\ln(h + 1) - \frac{30.25}{h+1} B^2 \right) \right] \frac{b}{a} =$$

$$\frac{\pi(10.5)^2}{4(144)} \left[\left((1 - 0.09658)^2 1 + 11 * 0.09658(1 - 0.09658)\ln(1 + 1) - \frac{30.25}{1+1} 0.09658^2 \right) - \left((1 - 0.09658)^2 0 + 11 * 0.09658(1 - 0.09658)\ln(0 + 1) - \frac{30.25}{0+1} 0.09658^2 \right) \right] = 0.97565 \text{ ft}^3$$

$$S_{visb} = \frac{\pi(10.5)^2}{4(144)} \left[\left((0.91882 - 0.08593)^2 1 + 11 * 0.08593(0.91882 - 0.08593)\ln(1 + 1) - \frac{30.25}{1+1} 0.08593^2 \right) - \left((0.91882 - 0.08593)^2 0 + 11 * 0.08593(0.91882 - 0.08593)\ln(0 + 1) - \frac{30.25}{0+1} 0.08593^2 \right) \right] = 0.81243 \text{ ft}^3$$

$$S_{odsw} = S_{visb} * SG_{gw} * W = 0.81243 * 0.35 * 62.4 = 17.74 \text{ lb}$$

$$S_{odsb} = (S_{vosb} - S_{visb})SG_{gb} * W = (0.97565 - 0.81243)0.5 * 62.4 = 5.09 \text{ lb}$$

$$S_{odt} = (S_{odsw} + S_{odsb})CRM_{AdjFac} = (17.74 + 5.09)0.76 = 17.47 \text{ lb}$$

6. Top and branch biomass (DRYBIO_TOP)

$$T_{odt} = (TAB - MST - STP - FOL) CRM_{AdjFac} = (612.75 - 444.87 - 22.84 - 12.90)0.76 = 101.09 \text{ lb}$$

7. Total aboveground biomass (CRM)

$$B_{odt} + S_{odt} + T_{odt} = 340.30 + 17.47 + 101.09 = 458.86 \text{ lbs}$$

8. Total aboveground carbon (CARBON_AB)

$$(B_{odt} + S_{odt} + T_{odt}) * 0.5 = (340.30 + 17.47 + 101.09)0.5 = 229.43 \text{ lb}$$

Example 3: Volume, biomass, and carbon calculations for a standing dead tree

Example tree information: live (STATUSCD = 2) rough cull (TREECLCD = 4) quaking aspen (SPCD = 746) sample tree that is 10.5 inches d.b.h.

1. Gross volume (VOLCFGRS)

$$V_1 = \text{top diameter outside bark} = 4 \text{ inches}$$

$$V_2 = \text{predicted bole length} = b_{12} + b_{13} \left(1 - e^{-b_{14}x_1} \right)^{b_{15}} x_2^{b_{16}} \left(b_{17} - \frac{V_1}{x_1} \right)^{b_{18}} x_3^{b_{19}} = 4.5 + 6.4301(1 - e^{-0.23545*10.5})^{1.338} * 73^{0.4737}(1.00001 - 4.5/10.5)^{0.73385} * 78.33^{0.8228} = 48.42 \text{ ft}$$

$$x_1 = \text{d.b.h.} = 10.5 \text{ inches}$$

$$x_2 = \text{site index} = 73 \text{ ft}$$

$$x_3 = \text{basal area of live trees on plot/condition} = 78.33 \text{ ft}^2$$

$$VOLCFGRS = (b_0 + b_1x_1 + b_2V_1 + b_3x_1^2 + b_4x_1^2V_2 + b_5V_2^2 + b_6V_2V_1^2 + b_7x_1^2V_2^3 + b_8x_1^2V_2^2V_1)b_9(b_{10} + b_{11}x_1) / 100 = (0.0030086 + 0.0020355*10.5 + -0.0030018*4$$

$$+ 0.000062381*10.5^2 + 0.000025705*10.5^2*48.42 + -0.000007009*48.42^2 + 0.000036708*48.42*4^2 + 0.00000000814*10.5^2*48.42^3 + -0.0000000019*10.5^2*48.42^2*4)79(91.51 + 0.37*10.5) / 100 = 13.32 \text{ ft}^3$$

2. Sound volume (VOLCFNSND)

$$\text{VOLCFNSND} = \text{VOLCFGRS}(1 - \frac{b_1 b_2}{100}) = 13.32(1 - \frac{61.8*0.75}{100}) = 7.14 \text{ ft}^3$$

3. Bole biomass (DRYBIO_BOLE)

$$\text{Bolewood biomass (eq. 1): } B_{\text{odw}} = V_{\text{gw}} * SG_{\text{gw}} * W = 7.14 * 0.35 * 62.40 = 155.94 \text{ lbs}$$

$$\text{Bole bark biomass (eq. 2): } B_{\text{odb}} = V_{\text{gw}} * BV\% * SG_{\text{gb}} * W = 7.14 * 0.144 * 0.5 * 62.4 = 32.10 \text{ lb}$$

$$\text{Total bole biomass (eq. 3): } B_{\text{odt}} = B_{\text{odw}} + B_{\text{odb}} = 155.92 + 32.10 = 188.12 \text{ lb}$$

4. Component ratio method adjustment factor (CRM_{AdjFac})

$$\text{CRM adjustment factor (eq.4): } \text{CRM}_{\text{AdjFac}} = \frac{B_{\text{odt}}}{\text{MST}} = \frac{188.12}{444.87} = 0.42$$

5. Stump volume and biomass (DRYBIO_STUMP)

$$\text{Stump volume (eq. 5): } S_{\text{vosb}} = \frac{\pi(abh)^2}{4(144)} \left[\left((A - B)^2 h + 11B(A - B)\ln(h + 1) - \frac{30.25}{h+1} B^2 \right) \right] \frac{b}{a} =$$

$$\frac{\pi(10.5)^2}{4(144)} \left[\left((1 - 0.09658)^2 1 + 11 * 0.09658(1 - 0.09658)\ln(1 + 1) - \frac{30.25}{1+1} 0.09658^2 \right) - \left((1 - 0.09658)^2 0 + 11 * 0.09658(1 - 0.09658)\ln(0 + 1) - \frac{30.25}{0+1} 0.09658^2 \right) \right] = 0.97565 \text{ ft}^3$$

$$S_{\text{visb}} = \frac{\pi(10.5)^2}{4(144)} \left[\left((0.91882 - 0.08593)^2 1 + 11 * 0.08593(0.91882 - 0.08593)\ln(1 + 1) - \frac{30.25}{1+1} 0.08593^2 \right) - \left((0.91882 - 0.08593)^2 0 + 11 * 0.08593(0.91882 - 0.08593)\ln(0 + 1) - \frac{30.25}{0+1} 0.08593^2 \right) \right] = 0.81243 \text{ ft}^3$$

$$\text{Stump wood biomass (eq. 6): } S_{\text{odsw}} = S_{\text{visb}} * SG_{\text{gw}} * W = 0.81243 * 0.35 * 62.4 = 17.74 \text{ lb}$$

$$\text{Stump bark biomass (eq. 7): } S_{\text{odsb}} = (S_{\text{vosb}} - S_{\text{visb}})SG_{\text{gb}} * W = (0.97565 - 0.81243)0.5 * 62.4 = 5.09 \text{ lb}$$

$$\text{Total stump biomass (eq. 8): } S_{\text{odt}} = (S_{\text{odsw}} + S_{\text{odsb}})CRM_{\text{AdjFac}} = (17.74 + 5.09)0.42 = 9.66 \text{ lb}$$

6. Top and branch biomass (DRYBIO_TOP)

$$\text{Top and branch biomass (eq. 9): } T_{\text{odt}} = (\text{TAB} - \text{MST} - \text{STP} - \text{FOL}) CRM_{\text{AdjFac}} = (612.75 - 444.87 - 22.84 - 12.90)0.42 = 55.88 \text{ lb}$$

7. Total aboveground biomass (CRM)

$$\text{Total aboveground biomass} = B_{\text{odt}} + S_{\text{odt}} + T_{\text{odt}} = 188.12 + 9.66 + 55.88 = 253.66 \text{ lbs}$$

8. Total aboveground carbon (CARBON_AB)

$$\text{Total aboveground carbon} = (B_{\text{odt}} + S_{\text{odt}} + T_{\text{odt}}) * 0.5 = (188.12 + 9.66 + 55.88)0.5 = 126.83 \text{ lb}$$

Example 4: Biomass and carbon calculations for a sapling or woodland species (< 5 inches in diameter)

Example tree information: a live (STATUSCD = 2) quaking aspen (SPCD = 746) sapling that is 4.5 inches d.b.h.

1. Biomass (eq. 10): $Sap_{\text{odt}} = (\text{TAB} - \text{FOL})Sap_{\text{adj}} = (81.10 - 1.71)0.69089 = 54.85 \text{ lb}$

CONCLUSION

This publication and accompanying electronic files on a CD-ROM present for the first time in one report FIA's volume models and methods to estimate volume, biomass, and carbon for trees in forests across the United States. Where applicable, emerging areas of research have been noted with continued refinement of FIA's modeling/estimation procedures expected in the future.

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Appendices Printed in this Report

Appendix A.—Gross and sound cubic-foot volume equation forms for the Northern, Southern, Rocky Mountain, and Pacific Northwest (excluding AK) regions.

Appendices and Accompanying Excel Files Included on CD-ROM

Appendix B.—Alaska gross cubic-foot volume equation forms

Appendix C.—Alaska sound cubic-foot volume algorithm

volcfsnd_eqn_coefs.xlsx

volcfgrs_eqn_coefs.xlsx

REF_SPECIES.xlsx

Selection of Previously Published Studies Cited in this Report and Included on CD-ROM

Publications cited in body of GTR:

Birdsey 1992: gtr_wo059

Heath et al. 2009: rmrs_p056_heath.pdf

Jenkins et al. 2004: ne_gtr319.pdf

Miles and Smith 2009: rn_nrs38.pdf

Raile 1982: rp_nc224.pdf

Volume equation publications:

Chittester and MacLean 1984: pnw_rn420.pdf

Curtis et al. 1968: pnw_rp056.pdf

Hahn 1984: rp_nc250.pdf

Hahn and Hansen 1991: Hahn_and_Hansen_1991.pdf

Larson and Winterberger 1988: pnw_rn478.pdf

Pillsbury and Kirkley 1984: pnw_rn414.pdf

Scott 1981: ne_rn304.pdf

APPENDIX A

Gross cubic-foot volume equation forms for the Northern, Southern, Rocky Mountain, and Pacific Northwest (excluding AK) regions. (dbh = diameter at breast height, drc = diameter at root collar, ht = total height)

Table 1.—Gross cubic-foot volume (VOLCFGRS) equation forms for the Northern region

Region/States (species code ^a)	Form of the model: VOLCFGRS = $f(x_1, x_2, \dots, x_n)$	Observed items: (x_1, x_2, \dots, x_n)	Reference ^b
North Central - IA, IL, IN, MO (all species except 66) KS, ND, NE, SD (all species except 66 and 122)	IF $x_2 = \text{NULL}$ VOLCFGRS = NULL OTHERWISE VOLCFGRS = $b_1 x_2^{b_2} (1 - e^{b_3 x_1^{b_4}})$ (IF VOLCFGRS < 0, SET TO 0)	$x_1 = \text{dbh}$ $x_2 = \text{site index}$	Hahn and Hansen 1991
North Central - MI, MN, WI (all species except 66)	IF $x_2 = \text{NULL}$ OR $x_3 = \text{NULL}$ VOLCFGRS = NULL OTHERWISE VOLCFGRS = $(b_0 + b_1 x_1 + b_2 V_1 + b_3 x_1^2 + b_4 x_1^2 V_2 + b_5 V_2^2 + b_6 V_2 V_1^2 + b_7 x_1^2 V_2^3 + b_8 x_1^2 V_2^2 V_1) b_9 (b_{10} + b_{11} x_1) / 100$, Where $V_1 = \text{top diameter outside bark} = 4$ And $V_2 = \text{predicted bole length} = b_{12} + b_{13} (1 - e^{-b_{14} x_1})^{b_{15}} x_2^{b_{16}} (b_{17} - \frac{V_1}{x_1})^{b_{18}} x_3^{b_{19}}$ (IF VOLCFGRS < 0, SET TO 0)	$x_1 = \text{dbh}$ $x_2 = \text{site index};$ if $x_2 < 20$, set to 20; if $x_2 > 120$, set to 120 $x_3 = \text{basal area of live trees on plot/condition};$ if $x_3 < 50$, set to 50; if $x_3 > 350$, set to 350	Hahn 1984
North Central, All States (66)	IF $V_1 \leq b_3$ VOLCFGRS = $b_0 + b_1 V_1 + b_2 V_1^2$ OTHERWISE VOLCFGRS = $b_0 + b_1 V_1 + b_2 (3b_3^2 - (2b_3^3/V_1))$, Where $V_1 = x_1^2 x_2 10^{-3}$ (IF VOLCFGRS <= 0, SET TO 0.1)	$x_1 = \text{drc}$ $x_2 = \text{ht}$	Chojnacky 1994
North Central - KS, ND, NE, SD (122)	IF $x_1^2 x_2 \leq b_1$ VOLCFGRS = $b_2 + b_3 x_1^2 x_2$ IF $x_1^2 x_2 > b_1$ VOLCFGRS = $b_4 + b_5 x_1^2 x_2$ (IF VOLCFGRS <= 0, SET TO 0.1)	$x_1 = \text{dbh}$ $x_2 = \text{ht}$	Myers 1964b
Northeast – All States (all species)	IF $x_2 = \text{NULL}$ VOLCFGRS = NULL OTHERWISE VOLCFGRS = $b_0 + b_1 x_1^{b_2} + b_3 x_1^{b_4} x_2^{b_5}$	$x_1 = \text{dbh}$ $x_2 = \text{bole height}$	Scott 1981

^a Species numbers refer to the codes used by FIA (Woudenberg et al. 2010)

^b References in Tables 1-4 of Appendix A are listed in a separate Literature Cited section at the end of the Appendix.

Table 2.—Gross cubic-foot volume (VOLCFGRS) equation forms for the Southern region

Region/States (species code ^a)	Form of the model: VOLCFGRS = f(x ₁ ,x ₂ ,...,x _n)	Observed items: (x ₁ ,x ₂ ,...,x _n)	Reference ^b
South, All States (All Species other than those listed below)	IF x₁ >= 5 AND x₂ ≠ NULL VOLCFGRS = b ₀ + b ₁ x ₁ ² x ₂ OTHERWISE VOLCFGRS = NULL	x ₁ = dbh x ₂ = ht	Unknown
South, All States (58, 59, 69, 106, 140, 141)	IF V₁ <= b₃ VOLCFGRS = b ₀ + b ₁ V ₁ + b ₂ V ₁ ² OTHERWISE VOLCFGRS = b ₀ + b ₁ V ₁ + b ₂ (3b ₃ ² - (2b ₃ ³ /V ₁)), Where V ₁ = x ₁ ² x ₂ 10 ⁻³ (IF VOLCFGRS <= 0, SET TO 0.1)	x ₁ = drc x ₂ = ht	Chojnacky 1994
South, All States (61, 63, 66, 303, 321, 755, 756, 758, 810, 843, 846, 867, 8513)	IF V₁ <= b₆ VOLCFGRS = b ₁ + b ₂ V ₁ + b ₃ V ₁ ² OTHERWISE VOLCFGRS = b ₄ + b ₂ V ₁ - (b ₅ /V ₁), Where V ₁ = x ₁ ² x ₂ 10 ⁻³ (IF VOLCFGRS <= 0, SET TO 0.1)	x ₁ = drc x ₂ = ht	Chojnacky 1988
South, All States (122)	IF x₁ < 21 VOLCFGRS = (b ₁ + b ₂ x ₁ ² x ₂) - (b ₃ + b ₄ $\frac{V_1^3 x_2}{x_1 b_5}$ + b ₆ x ₁ ²) Where V ₁ = top diameter outside bark = 4 OTHERWISE VOLCFGRS = (b ₇ + b ₈ x ₁ ² x ₂) - (b ₉ + b ₁₀ $\frac{V_1^3 x_2}{x_1 b_{11}}$ + b ₁₂ x ₁ ²) Where V ₁ = top diameter outside bark = 4 (If VOLCFGRS <= 0 AND x₁ >= 1, SET VOLCFGRS TO 0.1)	x ₁ = dbh x ₂ = ht	Hann and Bare 1978
South, All States (202)	VOLCFGRS = (b ₁ + b ₂ x ₁ ² x ₂) - (b ₃ + b ₄ $\frac{V_1^3 x_2}{x_1 b_5}$ + b ₆ x ₁ ²), Where V ₁ = top diameter outside bark = 4 (If VOLCFGRS <= 0 AND x₁ >= 1, SET VOLCFGRS TO 0.1)	x ₁ = dbh x ₂ = ht	Hann and Bare 1978

^a Species numbers refer to the codes used by FIA (Woudenberg et al. 2010)

^b References in Tables 1-4 of Appendix A are listed in a separate Literature Cited section at the end of the Appendix.

Table 3.—Gross cubic-foot volume (VOLCFGRS) equation forms for the Rocky Mountain region

Region/States (species code ^a)	Form of the model: VOLCFGRS = f(x ₁ ,x ₂ ,...,x _n)	Observed items: (x ₁ ,x ₂ ,...,x _n)	Reference ^b
AZ (All), NM (All), UT (South-West) (15, 17, 18, 19, 20, 21, 22, 93, 94, 96, 101, 102, 104, 108, 113, 114, 142, 202, 351, 352, 353, 374, 375, 746) RMRS, All States (103)	VOLCFGRS = (b ₁ + b ₂ x ₁ ² x ₂) - (b ₃ + b ₄ $\frac{V_1^3 x_2}{x_1 b_5}$ + b ₆ x ₁ ²), Where V ₁ = top diameter outside bark = 4 (IF VOLCFGRS <= 0 AND x₁ >= 1, SET VOLCFGRS TO 0.1)	x ₁ = dbh x ₂ = ht	Hann and Bare 1978
CO (All), NV, UT (North-East), WY (All) (15, 17, 18, 19, 20, 21, 22, 93, 94, 96, 101, 102, 108, 113, 114, 142, 202, 351, 352, 353, 374, 375, 746) CO (East), MT (East), WY (East) (51, 112, 116, 118, 122, 135, 137, 231) CO (All), NV, WY (All) (104)	IF x₁²x₂ <= b₅ VOLCFGRS = b ₁ + b ₂ x ₁ ² x ₂ IF x₁²x₂ > b₅ VOLCFGRS = b ₃ + b ₄ x ₁ ² x ₂ (IF VOLCFGRS <= 0 AND x₁ >= 1, SET VOLCFGRS TO 0.1)	x ₁ = dbh x ₂ = ht	Myers 1964a, Myers and Edminster 1972, Myers 1964b, Edminster et al. 1982
ID, MT (All) (Species 15, 17, 20, 21, 22, 72, 73, 101, 102, 104, 108, 113, 114, 142, 202) ID, MT (West) (51, 112, 116, 118, 122, 135, 137, 231) AZ (South), CO (West), ID, MT (All), NM (South), NV, WY (West) (117, 119)	VOLCFGRS = V ₁ - (V ₁ (b ₁ (($\frac{V_2}{b_2}$) ^{b₃} /x ₁ ^{b₄}))), Where V ₁ = b ₅ x ₁ ^{b₆} x ₂ ^{b₇} And V ₂ = top diameter outside bark = 4 (IF VOLCFGRS <= 0, SET TO 0.1)	x ₁ = dbh x ₂ = ht	Moisen unpublished (1989)
ID, MT (All) (18, 19, 81, 93, 94, 96, 351, 352, 353, 374, 375, 746) AZ (South), CO (All), ID, MT (All), NM (South), UT (All), WY (All) (242,263,264) RMRS, All States (741, 742, 745, 747, 748, 749)	IF x₁²x₂ <= b₅ OR (x₁ < 21 AND b₅ = 0) VOLCFGRS = b ₁ + b ₂ x ₁ ² x ₂ OTHERWISE VOLCFGRS = b ₃ + b ₄ x ₁ ² x ₂ (IF VOLCFGRS <= 0, SET TO 0.1)	x ₁ = dbh x ₂ = ht	Kemp (1956)

continued

Table 3.—continued

Region/States (species code ^a)	Form of the model: VOLCFGRS = f(x ₁ ,x ₂ ,...,x _n)	Observed items: (x ₁ ,x ₂ ,...,x _n)	Reference ^b
AZ (All), NM (All), UT (South-West) (51, 112, 116, 118, 122, 135, 137, 231)	<p>IF x₁ < 21 VOLCFGRS = (b₁ + b₂x₁²x₂) - (b₃ + b₄$\frac{V_1^3 x_2}{x_1^{b_5}}$ + b₆x₁²) Where V₁ = top diameter outside bark = 4</p> <p>OTHERWISE VOLCFGRS = (b₇ + b₈x₁²x₂) - (b₉ + b₁₀$\frac{V_1^3 x_2}{x_1^{b_{11}}}$ + b₁₂x₁²) Where V₁ = top diameter outside bark = 4</p> <p>(IF VOLCFGRS <= 0 AND x₁ >= 1, SET VOLCFGRS TO 0.1)</p>	x ₁ = dbh x ₂ = ht	Hann and Bare 1978
CO (West), NV, UT (North-East), WY (West) (51, 112, 116, 118, 122, 135, 137, 231)	<p>IF x₁ ≥ 5 VOLCFGRS = b₀ + b₁x₁²x₂</p> <p>OTHERWISE VOLCFGRS = NULL</p> <p>(IF VOLCFGRS <= 0 AND VOLCFGRS IS NOT NULL, SET TO 0.1)</p>	x ₁ = dbh x ₂ = ht	Edminster et al. 1980
RMRS, All States (Species 58, 59, 62, 63, 65, 66, 69, 106, 134, 140, 143)	<p>IF V₁ <= b₃ VOLCFGRS = b₀ + b₁V₁ + b₂V₁²</p> <p>OTHERWISE VOLCFGRS = b₀ + b₁V₁ + b₂(3b₃² - (2b₃³/V₁)), Where V₁ = x₁²x₂10⁻³</p> <p>(IF VOLCFGRS <= 0, SET TO 0.1)</p>	x ₁ = drc x ₂ = ht	Chojnacky 1994
ID, MT (All), NV (64)	<p>VOLCFGRS = ((V₁ + b₁) / (b₂ + b₃e^{b₄x₁})) + b₅, Where V₁ = b₆x₁²(b₇ + b₈x₂ - (b₉x₁x₂ / (x₂ + b₁₀))x₂(x₂ / (x₂ + b₁₀))² -- if V₁ <= 0, set V₁ to 2</p> <p>(IF VOLCFGRS <= 0, SET TO 1)</p>	x ₁ = MIN(dbh,60) x ₂ = ht	Chittester and MacLean 1984
RMRS, All States (68, 130, 136, 313, 361, 362, 404, 461, 462, 492, 544, 547, 552, 602, 606, 732, 823, 826, 901, 972, 974)	<p>IF (SPECIES < 300 AND x₁ < 9) OR (SPECIES >= 300 AND x₁ < 11) VOLCFGRS = b₁ + b₂x₁²x₂</p> <p>OTHERWISE VOLCFGRS = b₃ + b₄x₁²x₂</p> <p>(IF VOLCFGRS <= 0 OR x₂ = NULL OR x₁ < 5, SET VOLCFGRS TO 0.1)</p>	x ₁ = dbh x ₂ = ht	Unknown

continued

Table 3.—continued

Region/States (species code ^a)	Form of the model: VOLCFGRS = $f(x_1, x_2, \dots, x_n)$	Observed items: (x_1, x_2, \dots, x_n)	Reference ^b
RMRS, All States (133, 475)	<p>IF ($x_1 \geq 3$ AND $x_2 > 0$ AND $x_3 = 1$) VOLCFGRS = $(b_0 + b_1(x_1^2 x_2)^{b_2} + b_3)^3$ IF ($x_1 \geq 3$ AND $x_2 > 0$ AND $x_3 \neq 1$) VOLCFGRS = $(b_0 + b_1(x_1^2 x_2)^{b_2})^3$ OTHERWISE VOLCFGRS = 0.1 (IF VOLCFGRS ≤ 0, SET TO 0.1)</p>	<p>x_1 = drc x_2 = ht x_3 = # of stems</p>	<p>Chojnacky 1985</p>
<p>AZ (All), NM (All) (322, 814) RMRS, All States (756, 757, 758, 803, 810, 829, 843, 846, 847)</p>	<p>IF ($V_1 \leq b_6$) VOLCFGRS = $b_1 + b_2 V_1 + b_3 V_1^2$ OTHERWISE VOLCFGRS = $b_4 + b_2 V_1 - (b_5/V_1)$, Where $V_1 = x_1^2 x_2 10^{-3}$ (IF VOLCFGRS ≤ 0, SET TO 0.1)</p>	<p>x_1 = drc x_2 = ht</p>	<p>Chojnacky 1988</p>
<p>CO (All), ID, MT (All), NV, UT (All), WY (All) (322, 814)</p>	<p>IF ($x_1 \geq 3$ AND $x_2 > 0$ AND $x_3 = 1$) VOLCFGRS = $(b_0 + b_1(x_1^2 x_2)^{b_2} + b_3)^3$ IF ($x_1 \geq 3$ AND $x_2 > 0$ AND $x_3 \neq 1$) VOLCFGRS = $(b_0 + b_1(x_1^2 x_2)^{b_2})^3$ OTHERWISE VOLCFGRS = 0 (IF VOLCFGRS < 0, SET TO 0.1)</p>	<p>x_1 = drc x_2 = ht x_3 = # of stems</p>	<p>Chojnacky 1985</p>

^a Species numbers refer to the codes used by FIA (Woudenberg et al. 2010)

^b References in Tables 1-4 of Appendix A are listed in a separate Literature Cited section at the end of the Appendix.

Table 4.—Gross cubic-foot volume (VOLCFGRS) equation forms for the Pacific Northwest region

Region/States (species code ^a)	Form of the model: VOLCFGRS = f(x ₁ ,x ₂ ,...,x _n)	Observed items: (x ₁ ,x ₂ ,...,x _n)	Reference ^b
CA (All), OR (All), WA (All) (11, 42, 93, 98, 231, 242, 352) OR (East), WA (All) (15) OR (All), WA (All) (17, 19, 22, 73, 101, 108, 119) OR (East) (20) CA (All) (41, 251, 746, 747, 748, 760, 766, 768) WA (All) (72) CA (All), OR (All), WA (East) (92) OR (All), WA (West) (103) OR (West & JJ) (113, 130) OR (East & West) (341) CA (All), OR (East & JJ), WA (All) (351) CA (except Mixed-Conifer) (611)	$\text{VOLCFGRS} = ((10^{b_1} x_1^{b_2} x_2^{b_3} b_4) / ((b_5(1 + b_6 e^{b_7 \frac{x_1}{10}})) (b_8 x_1^2 + b_9) + b_{10})) ((b_{11} x_1^2 - b_9) / b_4)$	x ₁ = dbh x ₂ = ht	Brackett 1973, Browne 1962
CA (All) (Species 14, 19, 22) CA (All), OR (West & JJ) (20) CA (All), OR (All) (21)	$\text{VOLCFGRS} = b_1 x_1^2 x_2 V_1,$ Where $V_1 = b_2 + b_3 \frac{x_2}{x_1}$ -- if V₁ > 0.4, set to 0.4; if V₁ < 0.3, set to 0.3	x ₁ = MAX(dbh,6) x ₂ = ht	MacLean and Berger 1976
CA (All), OR (West & JJ) (15) CA (All) (17)	$\text{VOLCFGRS} = b_1 x_1^2 x_2 V_1,$ Where $V_1 = b_2 + b_3 x_2^{-1} + b_4 \frac{x_2^2}{x_1}$ -- if V₁ > 0.4, set to 0.4; if V₁ < 0.3, set to 0.3	x ₁ = MAX(dbh,6) x ₂ = ht	MacLean and Berger 1976
OR (All), WA (All) (41) CA (All) (50, 51, 54, 55) CA (except Mixed-Conifer) (52, 53) CA (All), OR (All), WA (All) (81)	$\text{VOLCFGRS} = b_1 x_1^2 x_2 V_1,$ Where $V_1 = b_2 + b_3 x_2^{-1}$ -- if V₁ < 0.27, set to 0.27	x ₁ = MAX(dbh,6) x ₂ = ht	MacLean and Berger 1976
CA (All), OR (All) (62, 65, 66) CA (All), OR (All), WA (All) (133, 321, 475)	IF (x₁ >= 3 AND x₂ > 0 AND x₃ = 1) $\text{VOLCFGRS} = (b_0 + b_1(x_1^2 x_2)^{b_2} + b_3)^3$ IF (x₁ >= 3 AND x₂ > 0 AND x₃ ≠ 1) $\text{VOLCFGRS} = (b_0 + b_1(x_1^2 x_2)^{b_2})^3$ OTHERWISE VOLCFGRS = 0.1 (IF VOLCFGRS <= 0, SET TO 0.1)	x ₁ = drc x ₂ = ht x ₃ = # of stems	Chojnacky 1985

continued

Table 4.—continued

Region/States (species code ^a)	Form of the model: VOLCFGRS = $f(x_1, x_2, \dots, x_n)$	Observed items: (x_1, x_2, \dots, x_n)	Reference ^b
CA (All), OR (All), WA (All) (64)	VOLCFGRS = $((V_1 + b_1) / (b_2 + b_3 e^{b_4 x_1}) + b_5,$ Where $V_1 = b_6 x_1^2 (b_7 + b_8 x_2 - (b_9 x_1 x_2 / (x_2 + b_{10})))$ $x_2 (x_2 / (x_2 + b_{10}))^2$ -- if $V_1 \leq 0$, set to 2 (IF VOLCFGRS ≤ 0, SET TO 1)	$x_1 =$ MIN(dbh,60) $x_2 =$ ht	Brackett 1973, Chittester and MacLean 1984
CA (All) (102, 103, 104, 108, 113, 124, 142) CA (All), OR (All) (120)	VOLCFGRS = $b_1 x_1^2 x_2 V_1,$ Where $V_1 = b_2 + b_3 \frac{x_2^2}{x_1}$ -- if $V_1 > 0.4$, set V_1 to 0.4; if $V_1 < 0.3$, set to 0.3	$x_1 =$ MAX(dbh,6) $x_2 =$ ht	MacLean and Berger 1976
CA (All) (101, 109, 119, 127, 137) CA (All), OR (West & JJ) (116, 122) CA (All), OR (All), WA (All) (117)	VOLCFGRS = $b_1 x_1^2 x_2 V_1,$ Where $V_1 = b_2 + b_3 x_1^{-1}$ -- if $V_1 > 0.4$, set to 0.4; if $V_1 < 0.3$, set to 0.3	$x_1 =$ MAX(dbh,6) $x_2 =$ ht	Brackett 1973, MacLean and Berger 1976
OR (East) (116) OR (East), WA (All) (122) OR (East), WA (East) (202)	VOLCFGRS = $((e^{(b_1 + b_2 \ln x_1 + b_3 (\ln x_2)^2 + \frac{b_4}{x_2^2} + b_5 \ln x_2)}$ $b_6) / ((b_7(1 + b_8 e^{(b_9 \frac{x_1}{10}))})(b_{10} x_1^2 + b_{11}) + b_{12}))((b_{13} x_1^2$ $- b_{11}) / b_6)$	$x_1 =$ dbh $x_2 =$ ht	Brackett 1973, Summerfield unpublished 1980, Browne 1962
CA (All) (201, 202)	VOLCFGRS = $b_1 x_1^2 x_2 V_1,$ Where $V_1 = b_2 + (b_3 \frac{x_2}{x_1} - b_4 \frac{x_2^2}{x_1})$ -- if $V_1 > 0.4$, set to 0.4; if $V_1 < 0.3$, set to 0.3	$x_1 =$ MAX(dbh,6) $x_2 =$ ht	MacLean and Berger 1976
OR (West & JJ), WA (West & CF) (202)	VOLCFGRS = $((10^{V_1} b_7) / ((b_8(1 + b_9 e^{(b_{10} \frac{x_1}{10}))})$ $(b_{11} x_1^2 + b_{12}) + b_{13}))((b_{14} x_1^2 - b_{12}) / b_7),$ where $V_1 =$ $b_1 + b_2 \log_{10} x_1 \log_{10} x_2 + b_3 (\log_{10} x_1)^2 +$ $b_4 \log_{10} x_1 + b_5 \log_{10} x_2 + b_6 (\log_{10} x_2)^2$	$x_1 =$ dbh $x_2 =$ ht	Brackett 1973, Browne 1962
CA (All), OR (All) (211, 212)	VOLCFGRS = $((e^{(b_1 + b_2 \ln x_1 + b_3 \ln x_2)} b_4) / ((b_5(1 +$ $b_6 e^{b_7 \frac{x_1}{10}})(b_8 x_1^2 + b_9) + b_{10}))((b_{11} x_1^2 - b_9) / b_4)$	$x_1 =$ dbh $x_2 =$ ht	Brackett 1973, Krumland and Wensel 1975, Browne 1962

continued

Table 4.—continued

Region/States (species code ^a)	Form of the model: VOLCFGRS = $f(x_1, x_2, \dots, x_n)$	Observed items: (x_1, x_2, \dots, x_n)	Reference ^b
CA (All), OR (All), WA (All) (263)	$\text{VOLCFGRS} = \left((10^{(b_1 + b_2 \log_{10} x_1 + b_3 \log_{10} x_2 + b_4 x_1)} b_5) / ((b_6(1 + b_7 e^{(b_8 \frac{x_1}{10})})) (b_9 x_1^2 + b_{10}) + b_{11}) \right) ((b_{12} x_1^2 - b_{10}) / b_5)$	$x_1 = \text{dbh}$ $x_2 = \text{ht}$	Brackett 1973, Chambers and Foltz 1979, Browne 1962
CA (All), OR (All), WA (All) (264, 299)	$\text{VOLCFGRS} = (b_1 x_1^{b_2} x_2^{b_3} b_4) / ((b_5(1 + b_6 e^{(b_7 \frac{x_1}{10})})) (b_8 x_1^2 + b_9) + b_{10}) ((b_{11} x_1^2 - b_9) / b_4)$	$x_1 = \text{dbh}$ $x_2 = \text{ht}$	Brackett 1973, Browne 1962, Bell et al. 1981
CA (All), OR (All) (312, 313, 361, 431, 540, 542, 815, 818, 901, 997) OR (West) (320) CA (All) (330, 333, 421, 492, 500, 600, 602, 660, 661, 730, 763, 801, 807, 811, 826, 839, 920, 998, 999) CA (All), OR (West & JJ) (631, 805, 981) CA (except Mixed-Conifer) (547, 604, 731, 922) CA (except Mixed-Conifer), OR (West), WA (East) (603) CA (All), OR (JJ) (821)	$\text{VOLCFGRS} = b_1 x_1^{b_2} x_2^{b_3}$	$x_1 = \text{dbh}$ $x_2 = \text{ht};$ if species 361 or 631, $x_2 = \text{MIN}(\text{ht}, 120)$	Pillsbury and Kirkley 1984
WA (All) (312, 313, 370, 431, 540, 542, 815, 901, 997) OR (West) (351, 926) WA (West & CF) (361) CA (All), OR (All), WA (All) (374, 375, 591) OR (All), WA (All) (492, 500, 602, 660, 661, 730, 746, 747, 760, 763, 766, 768, 920, 998, 999) OR (West & JJ) (510) OR (All), WA (West & CF) (600) OR (East), WA (West & CF) (631) WA (East) (818)	$\text{VOLCFGRS} = V_1 ((b_1 x_1^2 - b_2) / b_3),$ <p>Where $V_1 = (b_4 x_1^2 (x_2 + b_5) (b_6 V_2 + b_7 V_2 x_1 10^{-3} + b_8 V_2 x_2 10^{-3} + b_9 V_2 x_2 x_1 10^{-5} + b_{10} V_2 x_2^2 10^{-6} + b_{11} V_2 \sqrt{x_2} 10^{-3} + b_{12} V_3^4 x_1 10^{-3} + b_{13} V_3^4 x_2 10^{-3} + b_{14} V_3^{33} x_2 x_1 10^{-6} + b_{15} V_3^{33} \sqrt{x_2} 10^{-4} + b_{16} V_3^{41} x_2^2 10^{-7})$</p> $b_3) / ((b_{17} + b_{18} b_{19}^{(x_1 + b_{20})}) ((b_{21} (1 + b_{22} e^{(b_{23} \frac{x_1}{10})})) (b_1 x_1^2 + b_2) + b_{24})),$ <p>Where $V_2 = V_3^{2.5}$, And $V_3 = (x_2 + b_{25} - \frac{x_1}{b_{26}}) / (x_2 + b_5)$ -- if $V_1 \leq 0$, set to .01</p>	$x_1 = \text{dbh}$ $x_2 = \text{MAX}(\text{ht}, 18)$	Brackett 1973, Curtis et al. 1968, Browne 1962

continued

Table 4.—continued

Region/States (species code ^a)	Form of the model: VOLCFGRS = $f(x_1, x_2, \dots, x_n)$	Observed items: (x_1, x_2, \dots, x_n)	Reference ^b
CA (All) (510) CA (except Mixed-Conifer) (511)	$\text{VOLCFGRS} = ((b_1 x_1^2 x_2 b_2) / ((b_3(1 + b_4 e^{(b_5 \frac{x_1}{10})})) (b_6 x_1^2 + b_7) + b_8)) ((b_9 x_1^2 - b_7) / b_2)$	$x_1 = \text{dbh}$ $x_2 = \text{ht}$	Brackett 1973, MacLean unpublished 1983, Browne 1962
CA (All) (756) CA (except Mixed-Conifer) (758) CA (All), OR (East), WA (East) (990)	<p>IF ($V_1 \leq b_6$)</p> $\text{VOLCFGRS} = b_1 + b_2 V_1 + b_3 V_1^2$ <p>Where $V_1 = x_1^2 x_2 10^{-3}$</p> <p>OTHERWISE</p> $\text{VOLCFGRS} = b_4 + b_2 V_1 - (b_5 / V_1),$ <p>Where $V_1 = x_1^2 x_2 10^{-3}$</p> <p>(IF VOLCFGRS ≤ 0, SET TO 0.1)</p>	$x_1 = \text{drc}$ $x_2 = \text{ht}$	Chojnacky 1988
AK (All) (All Species)	See Appendix B for algorithms.		

^a Species numbers refer to the codes used by FIA (Woudenberg et al. 2010)

^b References in Tables 1-4 of Appendix A are listed in a separate Literature Cited section at the end of the Appendix.

JJ: Refers to Jackson and Josephine Counties, OR

CF: Refers to silver-fir region of WA

Table 5.—Sound cubic-foot volume (VOLCFSND) equation forms for all regions

Region/States (species code ^a)	Form of the model: VOLCFSND = $f(x_1, x_2, \dots, x_n)$	Observed items: (x_1, x_2, \dots, x_n)
North Central - KS, MI, MN, ND, NE, SD, WI (All Species)	VOLCFSND = VOLCFGRS($1 - \frac{b_1 b_2}{100}$)	
North Central - IA, IL, IN, MO (All Species)	<p>IF $x_1 < b_2$ VOLCFSND = VOLCFGRS($1 - \frac{V_1 b_3}{100}$), where $V_1 = b_0 + b_1 x_1$ -- if $V_1 > 100$, set to 100; if $V_1 < 0$, set to 0 (if $V_1 b_3 > 98$, set to 100)</p> <p>IF $x_1 \geq b_2$ VOLCFSND = VOLCFGRS($1 - \frac{V_1 b_3}{100}$), where $V_1 = b_0 + b_1 b_2$ -- if $V_1 > 100$, set to 100; if $V_1 < 0$, set to 0 (if $V_1 b_3 > 98$, set to 100)</p> <p>OTHERWISE VOLCFSND = 0</p>	$x_1 = \text{dbh}$
Northeast – All States, Rocky Mountain – All States, Pacific Northwest – CA, OR, WA (All Species) South – All States (All Species)	VOLCFSND = VOLCFGRS($1 - \frac{x_1}{100}$)	$x_1 = \text{percent of cubic-foot volume that is rotten or missing;}$ if $x_1 = \text{NULL}$, set to 0; if $x_1 > 98$, set to 100
Pacific Northwest – AK (All Species)	See Appendix C for algorithm.	

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The U.S. Forest Service, Forest Inventory and Analysis (FIA) program uses numerous models and associated coefficients to estimate aboveground volume, biomass, and carbon for live and standing dead trees for most tree species in forests of the United States. The tree attribute models are coupled with FIA's national inventory of sampled trees to produce estimates of tree growing-stock volume, biomass, and carbon, which are available in the Forest Inventory and Analysis database (FIADB; <http://fiatools.fs.fed.us>). To address an increasing need for accurate and easy-to-use documentation of relevant tree attribute models, needed individual tree gross volume, sound volume, biomass (including components), and carbon models for species in the United States are compiled and described in this publication with accompanying electronic files on a CD-ROM included with the publication. This report describes models currently in use as of 2010. These models are subject to change as the FIADB and associated tree attribute models are improved.

KEY WORDS: biomass equations, forest carbon, dead tree biomass, Forest Inventory and Analysis, volume equations, sound cubic foot, gross cubic foot

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