Outline of statistical analysis

• Objective: Estimate the difference in NO\textsubscript{x} emissions (if any) between biodiesel blends and conventional diesel fuel

• Meta-analysis, reanalyzing original data from three studies
  – Durbin 2011 (86 observations)
  – Durbin 2013 (32 observations)
  – Karavalakis 2014 (193 observations)

• Used multiple statistical methods and cross-checked results
Analytical considerations

• Focus mainly on engine testing
• Focus mainly on B5-soy
• Make as few statistical assumptions as possible:
  – Analyze each blend level separately, don’t assume straight-line relationship between blend level and NO\textsubscript{x} emissions
  – Treat each combination of study, engine type and drive cycle as a separate experiment
Biodiesel effect on NOx

NO$_x$ emissions adjusted for study + engine + cycle (g/bhp-hr)
Samples are unevenly distributed

### Durbin 2011

<table>
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<tr>
<th>Year</th>
<th>Engine Type</th>
<th>CRUISE - 40mph</th>
<th>CRUISE - 50mph</th>
<th>FTP</th>
<th>UDDS</th>
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### Durbin 2013

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### Karavalakis 2014

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Mixed model

• A linear mixed-effects model (or “mixed model”) has been the standard statistical approach for this type of problem since the 1950’s

• Available in standard statistical packages such as R or SAS
Mixed model results

• Difference between conventional diesel and
  Soy B5       ~1%
  Soy B10      ~2%

• Is the fuel effect significant?
  ➡ Significance test gives a P value of $10^{-15}$
  Confidence level = 1-P = Greater than 99.99%
Key Result

• For B5-soy, all our statistical approaches yield approximately the same results:
  – Approximately 1% increase in NO\textsubscript{x} emissions, compared with conventional diesel
  – Highly statistically significant (confidence level ≥ 99%)
Additional checks

• Consultant obtained the same result ("approximately 1%", confidence level > 99.9999%)

• Initial inclusion of chassis test data – similar results in magnitude and significance