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SUPPLEMENTARY INFORMATION

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Sharing a quota on cumulative carbon emissions

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Supplementary Text

1. Methods

Data: Emissions data to 2012 are from the global CO_2 budget compiled by the Global Carbon Project (GCP)[1], based on data for FFI (fossil-fuel and industrial) emissions from the Carbon Dioxide Analysis and Information Centre (CDIAC)[2], extrapolated for the most recent two years with provisional energy data from British Petroleum (BP)[3,1]. Population and GDP data are from the International Energy Agency (IEA)[4], currently available to 2011, with 1-year linear extrapolation to 2012. GDP data are by Purchasing Power Parity (PPP).

Regions: The 10 regions in Figures 1 to 3 (main paper) are the 10 subregions used in Representative Concentration Pathway scenarios[5]. Basic data are given in Supplementary Figure 1. Countries in Figure 4 (main paper) and Supplementary Figures 6 and 7 are the countries participating in the Deep Decarbonisation Pathways Project (DDPP)[6,7] of the United Nations Sustainable Development Solutions Network (SDSN), plus Ethiopia as a representative least-developed country.

Calculations: Shares (Figure 1, main paper) are calculated using Eq. (2) (main paper) with emissions (f_j) averaged over last 5 years of data, and population (p_j) averaged over a 5-year period centred on the time at which world population reaches 9 billion. Available quotas for CO_2 emissions from fossil fuels (Figure 2, main paper) are calculated with a global Q^{Avail} from Eq. (3) (main paper), using values given after Eq. (3) (main paper). Mitigation rates (Figure 3, main paper) are calculated with Eq. (4) (main paper) and Eq. (S10) (Supplementary Text 3), using initial emissions f_0 at 2012.5 and growth rates r averaged over 2008.5-2012.5. Unless otherwise specified, calculations use the following reference parameters: sharing index $w = 0.5$, all-time carbon quota $Q^{\mathrm{Tot}} = 3500$ $GtCO_2$, attribution start time and mitigation start time = 2013.0, no delay in starting mitigation.

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Committed emissions: We estimate committed emissions for each country and region according to the remaining expected lifetime of energy infrastructure (CO_2-emitting devices) existing in that country or region as of the end of 2013. Our analysis follows the methods detailed elsewhere[8,9], and includes commitments related to main activity power plants, "autoproducer" generators, road transport, train and air transport, and residential, industrial and commercial sectors.

Restrictions: This work has focussed on the sharing of an available carbon quota for CO_2 emissions from fossil fuels, exogenously allowing for emissions from land use change with Eq. (3) (main paper). Emissions of non-CO_2 gases and other radiatively active agents such as aerosols are not treated explicitly in sharing, but their effects are included in estimates of global carbon quotas consistent with specified warming limits[10].

2. *Properties and generalisations of blended sharing principles*

Equation (2) in the main paper can be rewritten as

$$s_j(w) = (1-w)\frac{f_j}{F} + w\frac{p_j}{P} = \frac{p_j}{P}\left[(1-w)\frac{f_j/p_j}{F/P} + w\right] \qquad \text{(S1)}$$

This demonstrates that the share s_j to a region j to increasing w hinges on whether its per-capita emission (f_j/p_j) is less than or greater than the global average per-capita emission (F/P). When $f_j/p_j = F/P$, the share s_j is independent of w. This criterion defines the pivot point for whether or not a country or region is favoured in its share by increasing equity in the sharing principle.

Equation (2) in the main paper can be generalised to include multiple metrics. For example, in addition to the distributions of emissions (reflecting inertia) and population (reflecting equity), the distribution of Gross Domestic Product (GDP) can be used as a metric of capacity for mitigation. This would suggest the three sharing principles

$$s_j(\text{Emis}) = \frac{f_j}{F}; \quad s_j(\text{GDP}) = \frac{g_j}{G}; \quad s_j(\text{Pop}) = \frac{p_j}{P} \qquad \text{(S2)}$$

where s_j is the share of the quota to country j; f_j, g_j and p_j are respectively the emissions, GDP and population for country j; and F, G and P are emissions, GDP and the population for the world. Shares sum to 1 over all countries ($\sum s_j = 1$) because $\sum f_j = F$, $\sum g_j = G$ and $\sum p_j = P$. The resulting blended sharing principle is

$$s_j(\text{Blend}) = w_F \frac{f_j}{F} + w_G \frac{g_j}{G} + w_P \frac{p_j}{P} \tag{S3}$$

where w_F, w_G and w_P are weights that sum to 1 ($w_F + w_G + w_P = 1$), determining how the blended sharing principle combines each option in Eq. (S2). Equation (S3) satisfies $\sum s_j = 1$. Equation (S3) can be rewritten as

$$s_j(w_P, r_{FG}) = (1 - w_p)\left(r_{FG} \frac{f_j}{F} + (1 - r_{FG}) \frac{g_j}{G} \right) + w_P \frac{p_j}{P} \tag{S4}$$

where $r_{FG} = w_F/(w_F + w_G)$ (between 0 and 1) determines the relative weighting between emissions and GDP in the sharing principle. Because of the constraint $w_F + w_G + w_P = 1$, there are two free parameters in Eq. (S3); these are taken in Eq. (S4) as w_P and r_{FG}..

As with Eq. (S1), the multiple-metric sharing principle of Eq. (S3) can be rewritten as

$$\begin{aligned} s_j(\text{Blend}) &= w_F \frac{f_j}{F} + w_G \frac{g_j}{G} + w_P \frac{p_j}{P} \\ &= \frac{p_j}{P}\left[w_F \frac{f_j/p_j}{F/P} + w_G \frac{g_j/p_j}{G/P} + w_P \right] \end{aligned} \tag{S5}$$

This demonstrates that the share s_j to a region j increases with increasing w_F when its per-capita emission (f_j/p_j) is greater than the global average per-capita emission (F/P), and increases with increasing w_G when its per-capita GDP (g_j/p_j) is greater than the global average per-capita GDP (G/P). These criteria defines the pivot points for whether or not a country or region is favoured in its share by increasing the weights w_F and w_G in the sharing principle, subject to the constraint $w_F + w_G + w_P = 1$.

3. Capped emissions trajectories

A capped emission trajectory with mitigation starting at initial time $t = 0$ is modelled as $f(t) = f_0\, g_G(t)\, g_M(t)$, where f_0 is the initial emission (Supplementary Figure 4). Dimensionless multipliers $g_G(t)$ and $g_M(t)$ account for growth and mitigation, respectively, with $g_G(0) = 1$ and $g_M(0) = 1$. We take $g_G(t)$ to be linear in t and $g_M(t)$ to be a declining exponential in t at rate m, the applied mitigation rate. Specified initial conditions at $t = 0$ are f_0 and the initial proportional growth rate $r = (f^{-1}\mathrm{d}f/\mathrm{d}t)_{t=0}$. These constraints yield Eq. (4) in the main paper:

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$$f(t) = f_0\left(1 + (r+m)t\right)\exp(-mt) \tag{S6}$$

The cumulative emission is:

$$q(m) = \int_0^\infty f(t)\,dt = \frac{f_0(2m+r)}{m^2} \tag{S7}$$

The applied mitigation rate m required to meet a specified quota q is given by solving for m:

$$m(q) = \frac{1+\sqrt{1+rq/f_0}}{q/f_0} = \frac{1+\sqrt{1+rT}}{T} \tag{S8}$$

where $T = q/f_0$ is the emission time defined by the quota q and the initial emission f_0. When r is positive, $m > 2/T$.

If there is a delay time t_d in starting mitigation, the emissions trajectory becomes:

$$f(t) = \begin{cases} f_0 \exp(rt) & \text{for } t \le t_d \\ f_d\left(1 + (r+m)(t-t_d)\right)\exp\left(-m(t-t_d)\right) & \text{for } t > t_d \end{cases} \tag{S9}$$

where $f_d = f_0(\exp(rt_d)$ is the emission at $t = t_d$ (Supplementary Figure 4). The applied mitigation rate becomes:

$$m(q) = \left(1 + \sqrt{1 + \frac{m(q-q_d)}{f_0}}\right)\frac{f_0}{q-q_d} \tag{S10}$$

where $q_d = (f_0/r)(\exp(rt_d) - 1)$ is the unmitigated cumulative emission from $t = 0$ to $t = t_d$.

The applied mitigation rate m is different from the realised mitigation rate $m_R(t)$, the proportional rate at which $f(t)$ decreases at time t after accounting for initial emissions growth. The realised mitigation rate is

$$m_R(t) = -\frac{(\mathrm{d}f/\mathrm{d}t)(t)}{f(t)} = m - \frac{m+r}{1+(m+r)t} \tag{S11}$$

At $t = 0$, $m_R = -r$ (there is no realised mitigation). As t increases, m_R progressively approaches the applied rate m.

The emissions trajectory $f(t)$ satisfies the differential equation

$$f'(t) = \frac{\mathrm{d}f}{\mathrm{d}t} = \frac{m+r}{1+(m+r)t} f(t) - mf(t) \tag{S12}$$

with initial condition $f(0) = f_0$. The second term on the right hand side accounts for mitigation at the applied rate m, and the first term accounts for initial emissions growth at the proportional growth rate r. The effect of the first term decays with time. At $t = 0$, the two terms combine to yield $(f^{-1}\mathrm{d}f/\mathrm{d}t)_{t=0} = r$.

Table S1 summarises further mathematical properties of the capped emissions trajectory, including first and second derivatives, time of maximum emissions, time of maximum emissions decline (the inflection point in $f(t)$), the time at which emissions recover to the initial value f_0 as they decline, and the time at which emissions decline to half of the initial value f_0.

4. Attribution of historic emissions

Inclusion of historic responsibility for climate change in a principle for sharing the mitigation challenge (the "Brazilian proposal") involves defining an attribution start date in the past, and then sharing cumulative global emissions from that time onward rather than from the present. Let Q^{Hist} be the global historic cumulative emission to be included in the sharing principle (for example, the cumulative emission from 1750 to the end of 2012, the reference time for the start of mitigation in this work), and let the contribution of country or region j to this global total be q_j^{Hist}. Therefore, $\sum q_j^{\mathrm{Hist}} = Q^{\mathrm{Hist}}$. With an available global quota from the start of mitigation of Q^{Avail}, the global cumulative emission that is shared under this principle is $Q^{\mathrm{Avail}} + Q^{\mathrm{Hist}}$, and the share of this quota to country or region j is $s_j(Q^{\mathrm{Avail}} + Q^{\mathrm{Hist}})$, where s_j is the fractional share from Eq. (S1). For country or region j, its historic cumulative emission must be deducted from this overall quota to obtain its future available quota:

$$q_j^{\mathrm{Avail}} = s_j\left(Q^{\mathrm{Avail}} + Q^{\mathrm{Hist}}\right) - q_j^{\mathrm{Hist}} \tag{S13}$$

Summation over the world (with $\sum s_j = 1$) verifies that $\sum q_j^{\mathrm{Avail}} = Q^{\mathrm{Avail}}$. A shift to this sharing principle has no effect on required global mitigation rates, but it has large effects for regions and countries through the distribution of historic cumulative emissions q_j^{Hist} (Supplementary Figure 8).

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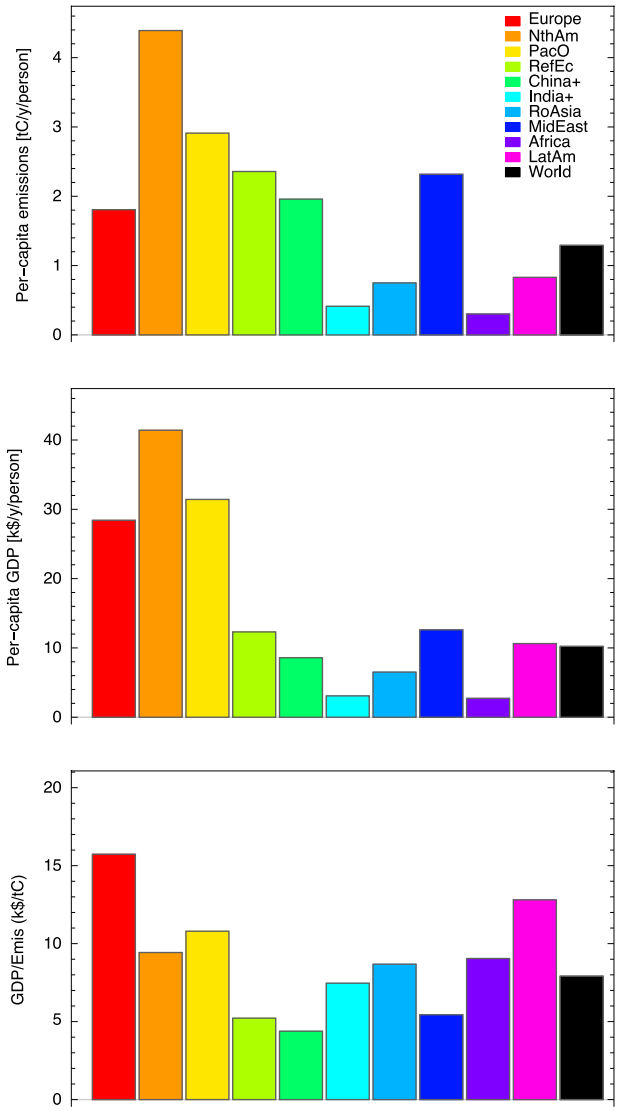
Supplementary Table

Emissions trajectory	$f(t) = f_0\left(1 + (r+m)t\right)\exp(-mt)$
Mitigation rate needed to meet quota q from t=0 onward (with emission time $T = q/f_0$)	$m(q) = \dfrac{1+\sqrt{1+rq/f_0}}{q/f_0} = \dfrac{1+\sqrt{1+rT}}{T}$
First derivative	$\dfrac{\mathrm{d}f}{\mathrm{d}t} = f'(t) = f_0\left(r - (r+m)mt\right)\exp(-mt)$
Second derivative	$\dfrac{\mathrm{d}^2 f}{\mathrm{d}t^2} = f''(t) = f_0 m\left(-m-2r+(r+m)mt\right)\exp(-mt)$
Relative growth rate (RGR)	$\mathrm{RGR}(t) = \dfrac{f'(t)}{f(t)} = \dfrac{m+r}{1+(m+r)t} - m$
Time of maximum emissions (when $f'(t_{\mathrm{Max}}) = 0$)	$t_{\mathrm{Max}} = \dfrac{r}{m(m+r)}$
Time of maximum emissions decline (inflection point, when $f''(t_{\mathrm{Inf}}) = 0$)	$t_{\mathrm{Inf}} = \dfrac{m+2r}{m(m+r)}$
Approximate time at which emissions return to initial emissions ($f(t_{1.0}) = f_0$)	$t_{1.0} \approx \dfrac{m^2+3mr+4r^2-m(m+r)\exp\left(2r/(m+r)\right)}{mr(m+r)}$ (First Newton iteration from initial estimate $t_{1.0} \approx 2t_{\mathrm{Max}}$, with typical error 1-2% overestimation)
Approximate time at which emissions fall to half of initial emissions ($f(t_{0.5}) = 0.5f_0$)	$t_{0.5} \approx \dfrac{4r + m\left[3 - \exp\left((m+2r)/(m+r)\right)/2\right]}{m(m+r)}$ (First Newton iteration from initial estimate $t_{0.5} \approx t_{\mathrm{Inf}}$, with typical error 1-2% overestimation)

Table S1: Summary of mathematical properties of the capped emissions trajectory.

Supplementary Figures



Supplementary Figure 1: Per capita CO_2 emissions from fossil fuels and other industrial processes (FFI), per capita GDP (Gross Domestic Production by Purchasing Power Parity), and the emissions efficiency of GDP (GDP/Emissions), for 10 regions spanning the world, in 2012. See Supplementary Text 1 for details of regions and data sets.

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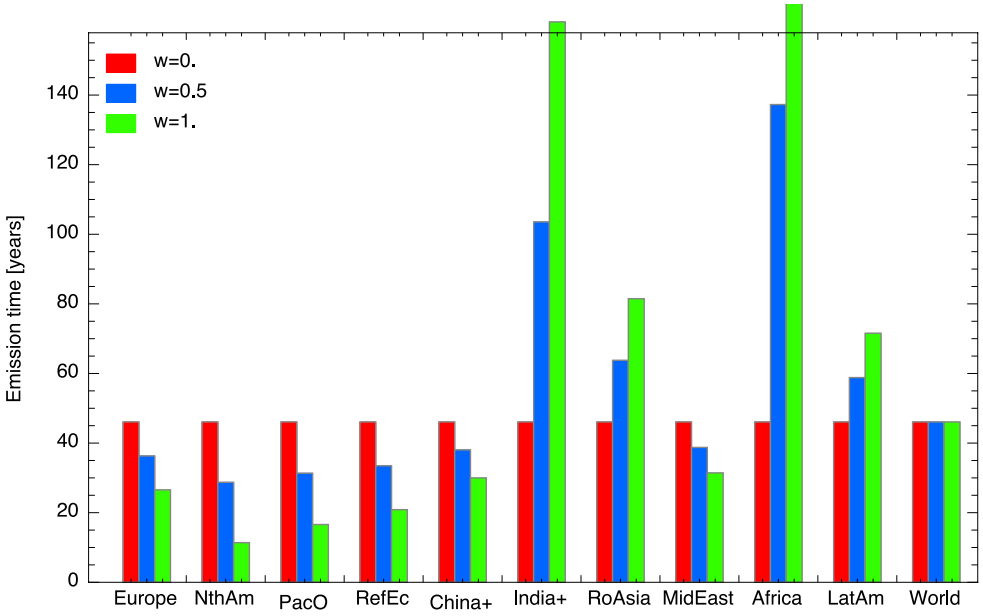
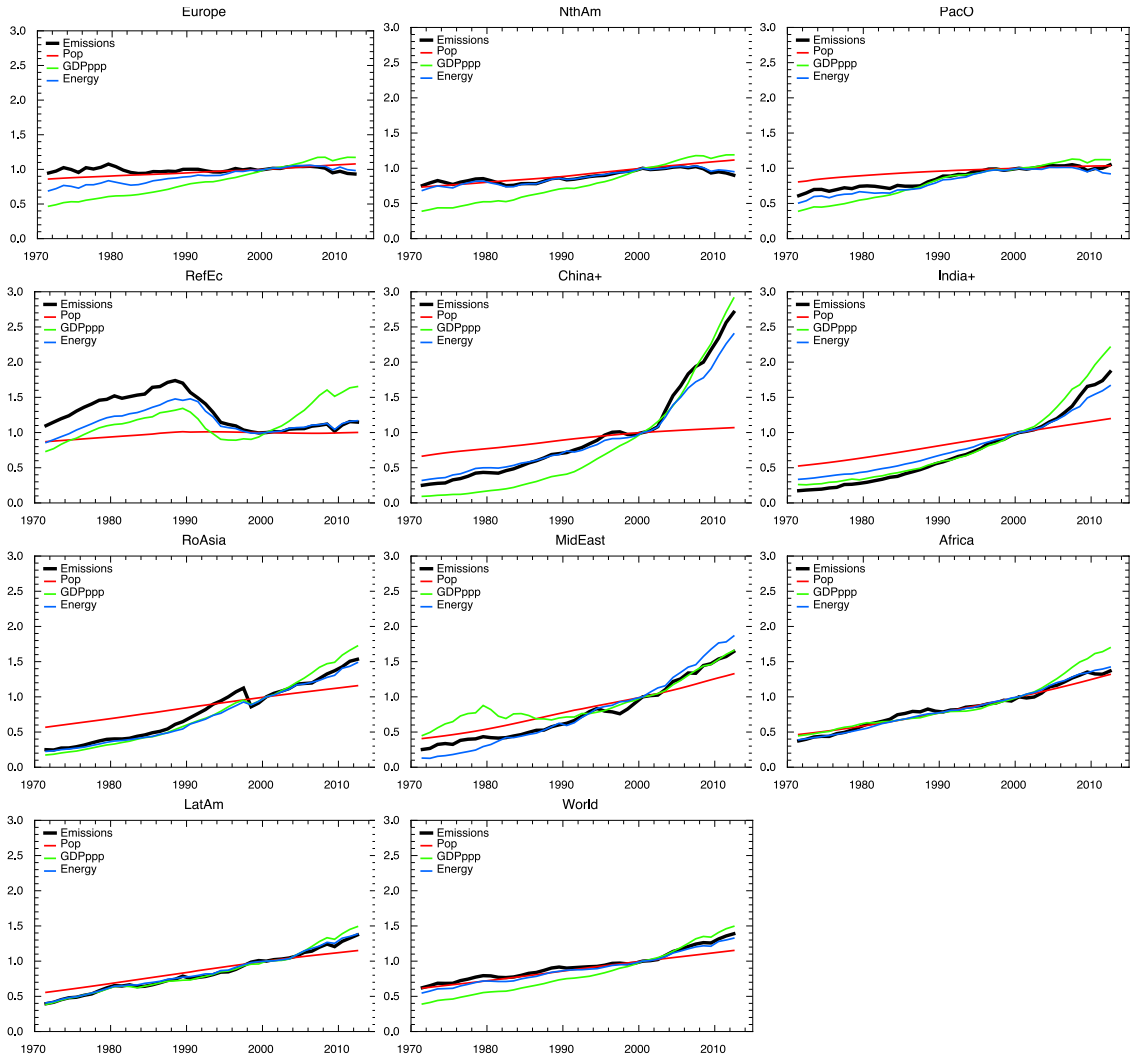


Supplementary Figure 2: Emission times for 10 regions and the world, showing the effect of varying sharing index (w = 0, 0.5, 1). Available carbon quota Q^{Avail} = 1400 $GtCO_2$, corresponding to a 2 degree warming limit at 50% success probability[10]. See Supplementary Text 1 for details of regions and parameters.



Supplementary Figure 3: Time series of normalised CO_2 emissions from fossil fuels and other industrial processes (FFI), population, GDP(ppp), and energy consumption, for 10 regions and the world. All quantities are normalised to 1 in 2000. See Supplementary Text 1 for details of regions and data sets.

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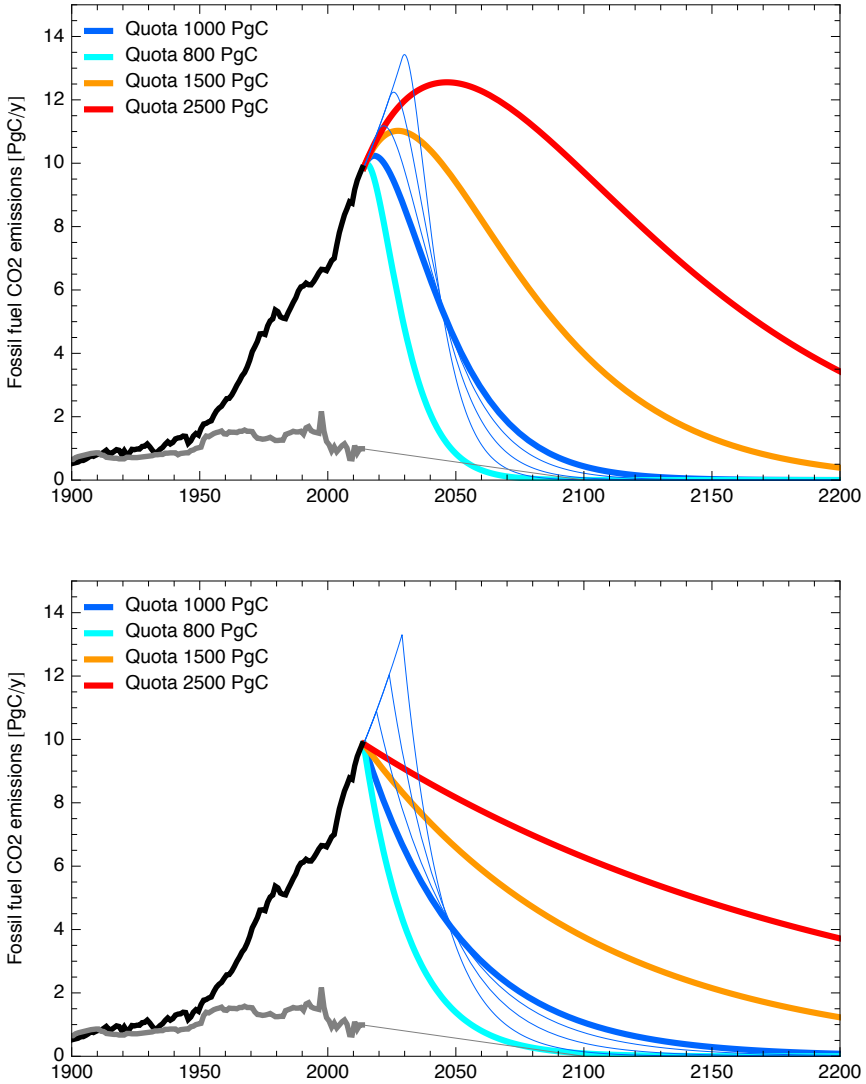
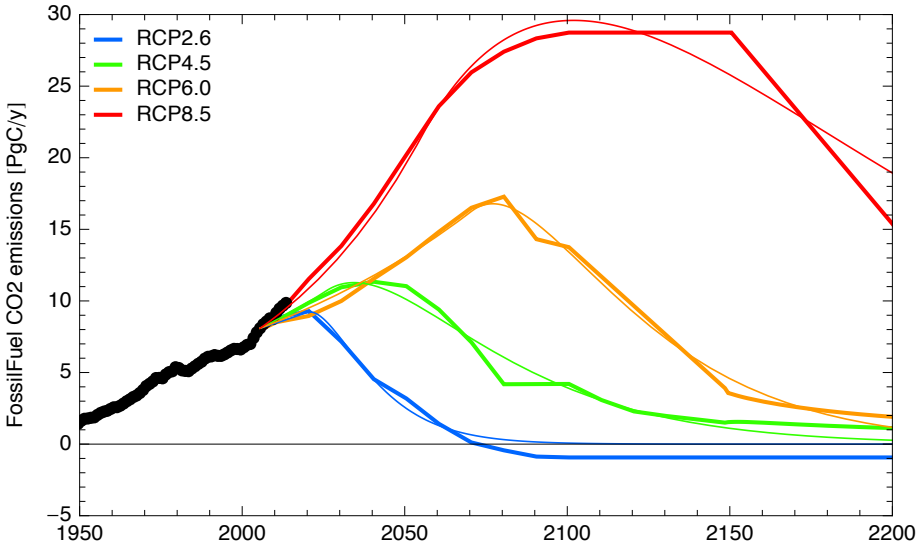


Supplementary Figure 4: (a, top): Global historic (black line) and future (thick coloured lines) trajectories for CO_2 emissions from fossil fuels and other industrial processes (FFI), with "smooth" future emissions trajectories that maintain continuity of emission growth rates at the break between past and future. formula follows Raupach et al 2011[11], Raupach 2013[12]. Grey line shows historic and assumed future trajectory for CO_2 emissions from land use change. Parameter is the all-time carbon quota Q_{Tot}, including past and future fossil-fuel and land use change emissions. Thin blue lines show the trajectories obtained if the start of mitigation is delayed by 5, 10 and 15 years. (b, bottom): As for (a), but with discontinuous emissions trajectories where exponential decline in emissions is assumed to begin immediately at the start of mitigation.



Supplementary Figure 5: Comparison of Representative Concentration Pathway (RCP) emission scenarios[13] (heavy lines) with analytic capped emissions trajectories with delay (light lines). Analytic trajectories were determined by fitting three parameters in Eq. (S9) (m, r, t_d) to each scenario. Equation (S9) captures the main features of the scenarios, except that it cannot account for negative emissions beyond 2070 in RCP2.6.

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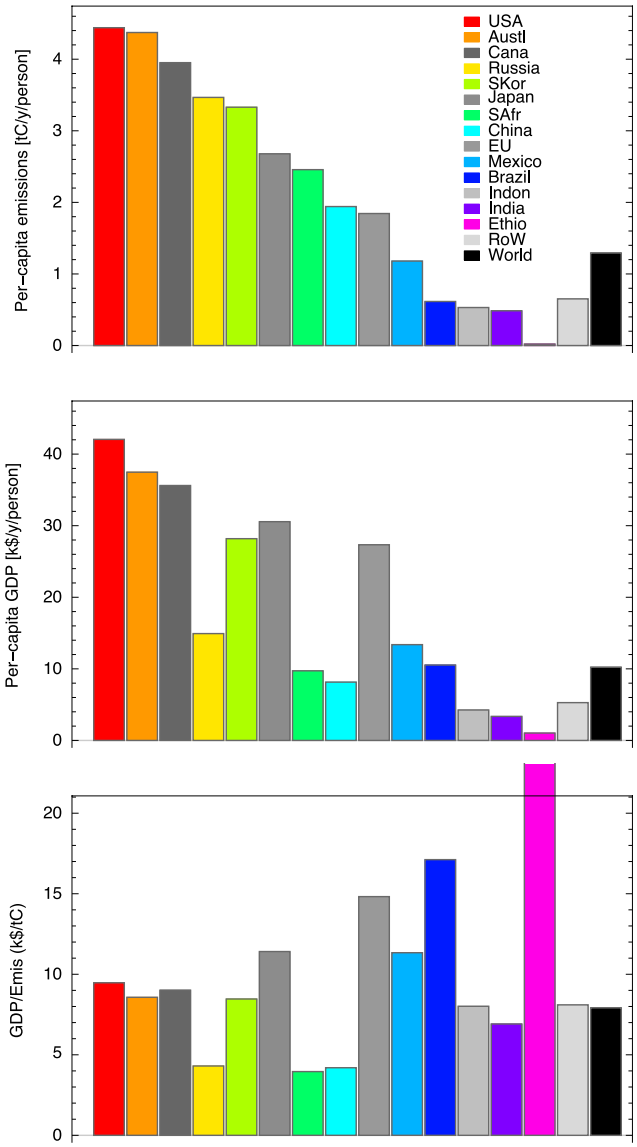
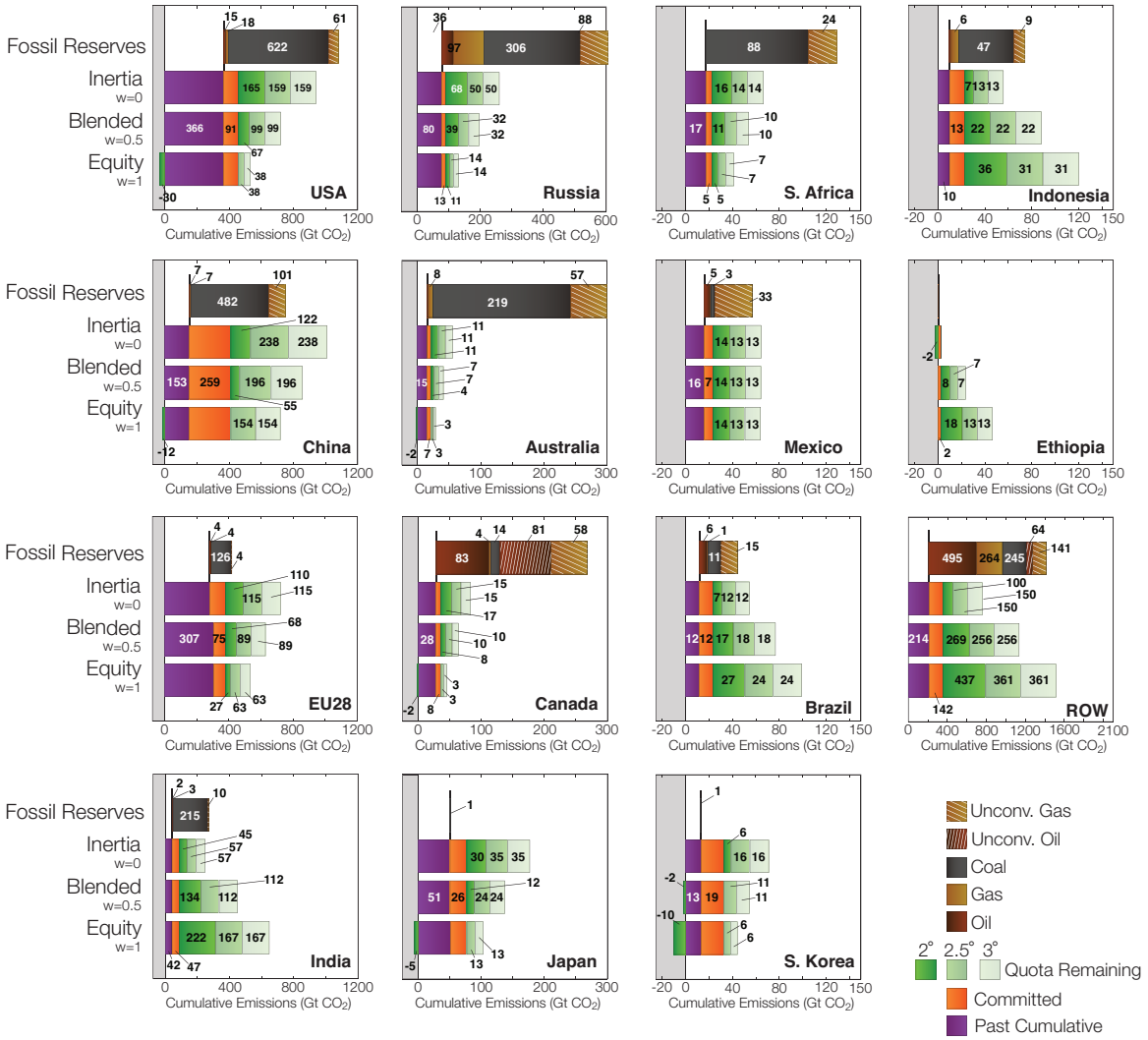


Supplementary Figure 6: Per capita CO_2 emissions from fossil fuels and other industrial processes (FFI) (top), per capita GDP (Gross Domestic Production by Purchasing Power Parity), and the emissions efficiency of GDP (GDP/Emissions), for 15 countries and regions, in 2012. See Supplementary Text 1 for details of regions and data sets. This figure is a national-level counterpart to the regional-level Supplementary Figure 1.



Supplementary Figure 7: Past cumulative CO_2 emissions from fossil fuels and other industrial processes (FFI) (purple), future committed emissions[8,14] (orange) and future quotas to meet warming limits of 2, 2.5 and 3 degrees with 50% probability (green), for 15 countries, under inertia, blended and equity sharing principles. Stacked bars are cumulative; numbers give the contribution of each increment to the quota (in $GtCO_2$). Negative increments are shown below the zero axis. Also shown are fossil-fuel reserves including coal, oil, gas, unconventional oil, unconventional gas[15] (not available for some countries). See Supplementary Text 1 for details of countries, regions and parameters. This figure is a national-level counterpart to the regional-level Figure 2 (main paper).

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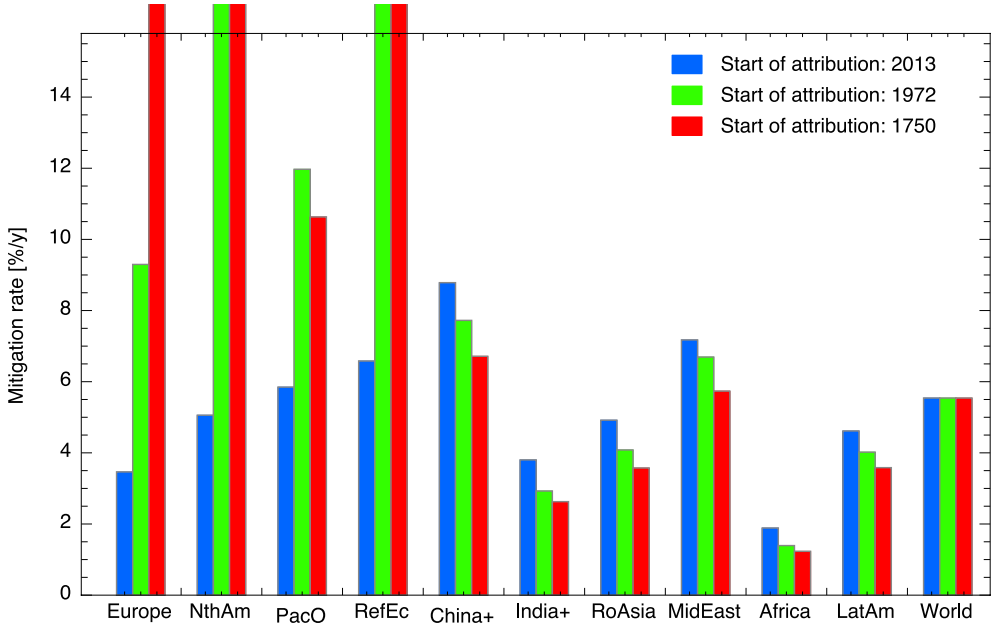
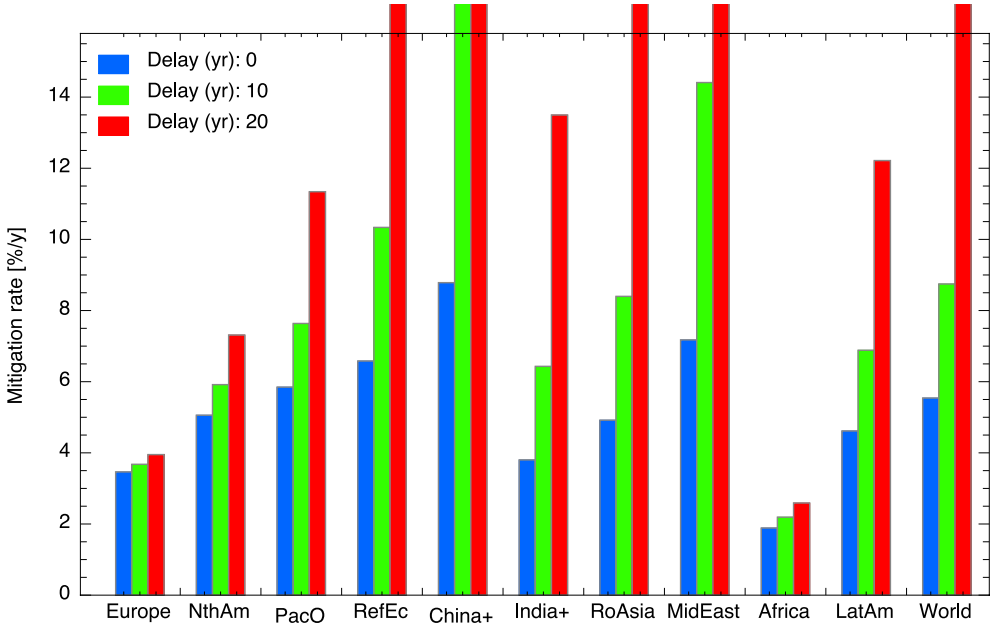


Supplementary Figure 8: Effect of attribution of historic emissions on mitigation rates for 10 regions and the world, with sharing index $w = 0.5$ and available carbon quota $Q^{\mathrm{Avail}} = 1400\ \mathrm{GtCO_2}$, corresponding to a 2 degree warming limit at 50% success probability[10]. See Supplementary Text 1 for details of regions and parameters.



Supplementary Figure 9: Effect of delay in starting mitigation on mitigation rates for 10 regions and the world, with sharing index $w = 0.5$ and available carbon quota $Q^{\mathrm{Avail}} = 1400$ $GtCO_2$, corresponding to a 2 degree warming limit at 50% success probability[10]. See Supplementary Text 1 for details of regions and parameters.

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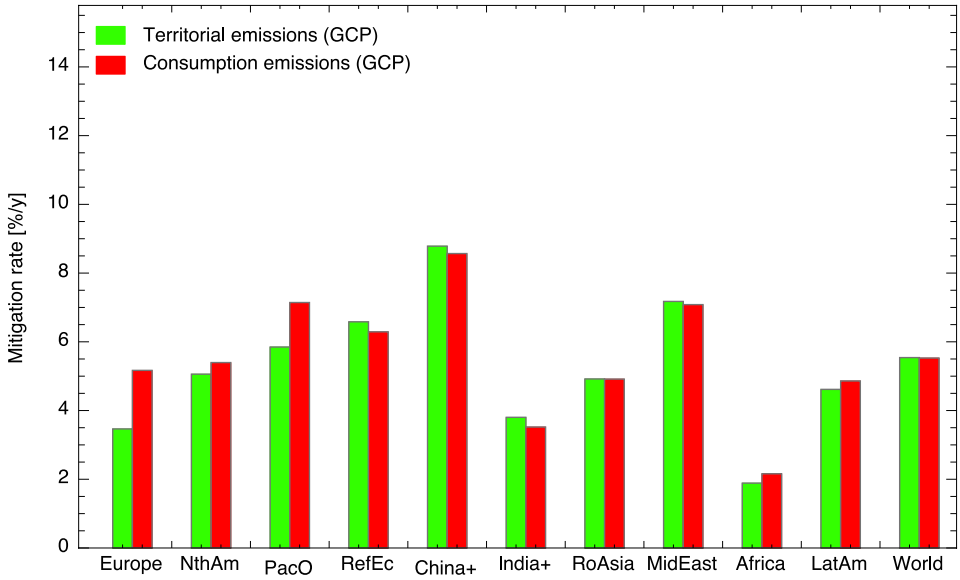
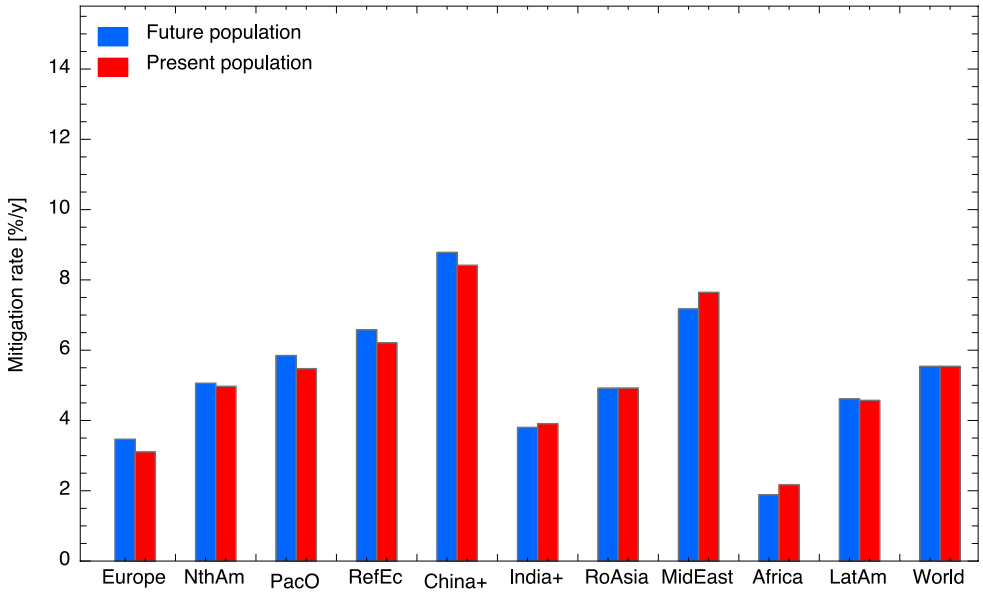


Supplementary Figure 10: Effect of using territorial (default) or consumption-based emissions on mitigation rates for 10 regions and the world, with sharing index $w = 0.5$ and available carbon quota $Q^{\mathrm{Avail}} = 1400\ \mathrm{GtCO_2}$, corresponding to a 2 degree warming limit at 50% success probability[10]. See Supplementary Text 1 for details of regions and parameters.



Supplementary Figure 11: Effect of using estimated population distribution with a global population of 9 billion (blue, default) or the present population distribution (red) on mitigation rates for 10 regions and the world, with sharing index $w = 0.5$ and available carbon quota $Q^{\mathrm{Avail}} = 1400$ $GtCO_2$, corresponding to a 2 degree warming limit at 50% success probability[10]. See Supplementary Text 1 for details of regions and parameters.

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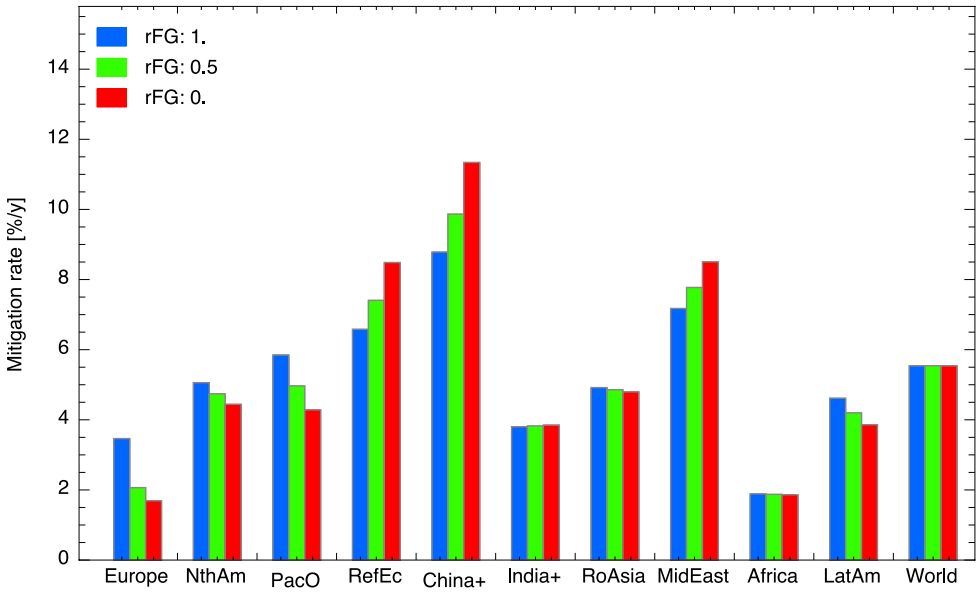
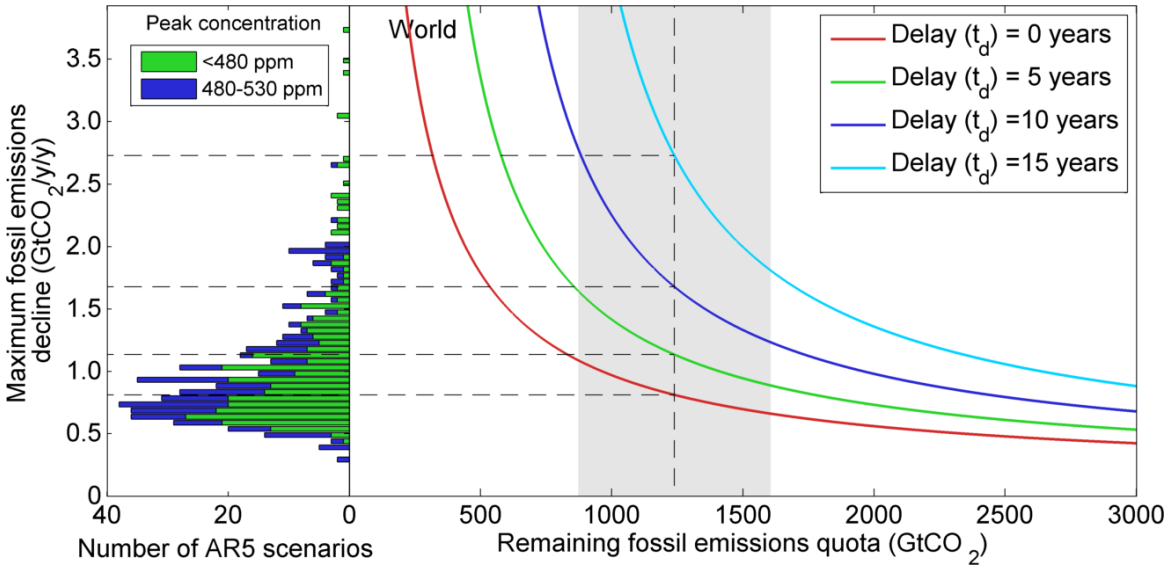


Supplementary Figure 12: Effect of including GDP as well as emissions in the sharing principle, for mitigation rates for 10 regions and the world. The parameter $r_{FG} = w_F/(w_F + w_G)$ (between 0 and 1) determines the relative weighting between emissions and GDP in the sharing principle, where w_F and w_G are respectively the weights for emissions and GDP; see Eq. (S4). The sharing index w (here 0.5) refers to the weighting between the combined emissions-GDP metric of "inertia" and population as a measure of "equity". See Supplementary Text 1 for details of regions and parameters.



Supplementary Figure 13: (left) Distribution of the maximum rate of emissions decline over 20-year period for the scenarios in the IPCC Fifth Assessment WG3 database that are consistent with peak CO_2 concentrations of less than 480 ppm (green) and 480-530 ppm (blue). (Right) maximum rate of emissions decline from the analytic capped-emissions trajectory incorporating delay, Eq. (S9), as a function of available quota Q^{Avail} (Eq. (S10) and Table 1). Both for scenarios peaking below 480 ppm and in the range 480-530 ppm, the median of the maximum rate of emissions decline is approximately consistent with the required maximum rate to achieve a quota of 1250 $GtCO_2$ if there is no delay in starting mitigation. Even a five-year delay causes the required rate to approach the upper edge of the distribution of maximum rates from the scenarios.

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