## Appendix D. Estimate the Number of Tests to Verify NOx Control Technologies that Reduces NOx between 15 to 25 Percent

This appendix presents the calculations used to estimate the minimum number of tests required for verification of NOx control strategies. Staff based its calculations for NOx on the same statistical basis used for PM control. That is, the same level of confidence that is attained by 3 tests of a PM strategy that achieves a 25% reduction should also be attained prior to verification of NOx reduction.

In order to calculate the number of tests required for a given reduction level, staff first estimated the test-to-test variability. Factors contributing to test-to-test variability include, but are not limited to the condition of engine or vehicle, fuel, driver (for chassis testing), diesel emission control system, and analytical equipment. Staff used 10% as the variability from test-to-test, which is consistent with data gathered from chassis and engine dynamometer testing.

The criterion being used is that the minimum number of tests required to have a 95% probability of detecting the specified emission reduction at the 95% confidence level. The basic equation being used for the calculation is:

$$n \approx (Z_\alpha + Z_\beta)^2 [\sigma^2_1 + (1-\delta/100)^2 \cdot \sigma^2_2]/\delta^2 \qquad \text{(D-1)}$$

Where:

- $n$ = sample size in each group;
- $\delta$ = difference between baseline and controlled engine emission mean; expressed as a percent of the baseline emission value;
- $\sigma^2_1$ = squared standard deviation of baseline-engine emission data, expressed as a percent of the baseline emission value;
- $\sigma^2_2$ = squared standard deviation of controlled-engine emission data, expressed as a percent of the baseline emission value;
- $1-\alpha$ = confidence coefficient on comparison of means;
- $1-\beta$ = probability of detection of reduction;
- $Z_\alpha$ = normal distribution value corresponding to upper-tail probability of $\alpha$; and
- $Z_\beta$ = normal distribution value corresponding to upper-tail probability of $\beta$.

Note that $\sigma_1$, $\sigma_2$ and $\delta$, are expressed as percentages of the baseline emission. The parameter 'z' is tabulated under different names in statistics reference texts. It is the 'z' value corresponding to 'the tail area of the unit normal distribution' in Box, Hunter, and Hunter (1978). In the standard Mathematical Tables (CRC, 1968), 'z' is known as 'x", and the tail area is labeled '1-F(x)', where F(x) is the cumulative distribution function of a standardized normal random variable.

**Table D.1 Estimation of Number of Tests**.

| | | | |
|---|---|---|---|
| Emission reduction relative to baseline (certification) emission, $\delta$, % | 15 | 20 | 25 |
| Known measurement variability at specified emission reduction, $\sigma_2$, % | 10 | 10 | 10 |
| Known measurement variability for baseline engine, $\sigma_1$, % | 10 | 10 | 10 |
| $\alpha$ | 0.05 | 0.05 | 0.05 |
| $z_\alpha$ | 1.645 | 1.645 | 1.645 |
| $\beta$ | 0.05 | 0.05 | 0.05 |
| $z_\beta$ | 1.645 | 1.645 | 1.645 |
| Number of tests required, n | 9.6 | 5.4 | 3.45 |

Table D.1 summarizes the estimate of number of tests, based on the confidence coefficient (0.95) and probability of detection (0.95), and test-to-test variability of 10 percent. It appears the number of tests is highly dependent on the emission reduction relative to the baseline. As noted from Table D.1, approximately three tests are needed to detect a 25 percent difference from the baseline. Since the above calculation is an estimate for the number of tests, the number of tests is truncated to the nearest lower integer. In order to maintain the same statistical basis, staff proposed the number of tests to be five and nine, in order to detect emission difference at 15 to 20 percent, 20 to 25 percent, respectively.

References for Appendix D

Box, G. E. P., W. G. Hunter, and J. S. Hunter. *Statistics for Experimenters*. John Wiley & Sons, New York, NY. 1978.