

Appendix F

Turbulent kinetic energy tendency equations and the Monin-Obhukov length scale.

Flow in the surface layer is commonly described in a coordinate system aligned with the mean wind, with z as the vertical dimension, x aligned with the wind, and y at right angles to the wind direction in the horizontal. Winds in directions x , y , and z are given as u , v , and w . With time averages indicated with an overbar, the horizontal wind \overline{u} may be decomposed into a mean wind, \overline{u} , and instantaneous deviations from that mean, u' (the wind fluctuation in the direction of the mean wind). Similarly, the instantaneous vertical velocity is w' but \overline{w} , the mean vertical velocity, is taken to be zero over flat ground. The production of turbulent kinetic energy (TKE) due to shear in the mean wind is the time averaged product of the instantaneous velocity fluctuations in the horizontal and the vertical times the gradient in the mean horizontal wind:

$$\textit{Shear production / consumption} = \overline{(u' w')} \frac{\partial \overline{u}}{\partial z}$$

Production and consumption of TKE by buoyancy forces within the surface layer is expressed as:

$$\textit{Bouyant TKE production / consumption} = \frac{g \overline{(w' \theta'_v)}_s}{\overline{\theta}_v}$$

The TKE equation may be expressed in a non-dimensional form by multiplying all terms by:

$$\frac{kz}{u_*^3}$$

Assuming that surface fluxes are approximately equal to fluxes in the surface layer, the buoyancy term in this form becomes:

$$\frac{kz g \overline{(w' \theta'_v)}_s}{\overline{\theta}_v u_*^3}$$

This buoyant production/consumption term from the non-dimensional TKE equation is commonly referred to with the symbol zeta ζ :

$$\zeta = \frac{k z g \overline{(w' \theta_v')}_s}{\overline{\theta_v} u_*^3}$$

The inverse of zeta multiplied by height z forms the Obhukov or Monin-Obhukov length scale, L, which is widely used to describe the stability of the surface layer.

$$\zeta = \frac{z}{L} = \frac{k z g \overline{(w' \theta_v')}_s}{\overline{\theta_v} u_*^3}$$

$$L = \frac{\overline{\theta_v} u_*^3}{k g \overline{(w' \theta_v')}}}$$